TESTS OF CAUSALITY
The Message in the Innovations

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I. Introduction

Recently, several authors have used new time series techniques to analyze the pairwise relationships between such macroeconomic variables as the rate of growth of the money supply (defined several ways) and the rate of inflation (Feige and Pearce, 1976), or the rate of growth of demand deposits and the treasury bill rate (Pierce, 1977a). No statistically significant relationship could be found between these variables. Because such findings are in conflict with most previous empirical work using similar data, they raise questions about the validity of the previous methodologies, the new time series techniques, or both.

This paper describes the new time series techniques and illustrates the advantages and disadvantages of these techniques relative to more traditional methods. Autoregressive-integrated-moving average (ARIMA) time series models are used to construct predictions of the variable based on the past history of the series. The residuals or prediction errors from the ARIMA model are estimates of the "innovations" of the series, the part of each observation which could not be predicted using past data. The innovations from one series are correlated with the innovations from another series at several leads and lags to determine the relationship between the variables. Several special cases are worked out to illustrate the advantages and shortcomings of this technique. I conclude that it is important to consider the *power* of this procedure before putting much faith in empirical results which seem to find a "lack of relationship" between macroeconomic time series variables.

II. A Definition of Causality

Suppose that there are time series observations available on two economic variables, $\{y_t\}$ and $\{x_t\}$, and there is a question about whether "y causes x," or "x causes y." For example, suppose that one questions whether the money supply "causes" nominal income, or vice versa.

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Granger (1969) suggests a definition of "causality" which is testable using regression or correlation techniques. Granger defines *simple causality* such that "x causes y" if knowledge of past x reduces the variance of the errors in forecasting y_t beyond the variance of the errors which would be made from knowledge of past y alone:

$$\sigma^2(y_t|y_{t-1},\ldots,x_{t-1},x_{t-2},\ldots) < \sigma^2(y_t|y_{t-1},\ldots).$$

Granger also defines *instantaneous causality*, where current as well as past values of x are used to predict y_t.[1] If y is related to current or lagged x, but not future x, x is *exogenous* relative to y. (This parallels the concept of statistical exogeneity which is assumed when least squares techniques are used to estimate distributed lag or linear regression models.)[2] If x causes y and y causes x, then there is *feedback* between the variables. If y does not cause x and x does not cause y (even instantaneously), the two series are *unrelated.*

Appendix A provides a more formal definition of causality in the context of a system of linear stochastic difference equations. For the purpose of this paper, consider the distributed lag model between current y and both current and past x,

$$y_t = a + \sum_{i=0}^{\infty} \beta_i x_{t-i} + \eta_t$$

$$= a + \beta(L)x_t + \eta_t \,, \qquad (1)$$

where $\beta(L)$ is a polynomial in the lag operator L, which is defined such that $L^k x_t \equiv x_{t-k}$. Sims (1972) proves that the disturbance η_t in (1) is uncorrelated with past, current, and future x if and only if "y does not cause x." If all of the coefficients of $\beta(L)$ are equal to zero, "x does not cause y."

Before considering time series methods of testing for causal relationships, it is worthwhile to consider the relationship of Granger causality to other

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[1] Pierce and Haugh (1977) prove that it is impossible to determine a unique direction of causality if instantaneous causality exists.

[2] However, this definition of exogeneity does not rule out the possibility of feedback between the variables, since instantaneous causality could exist. Nelson (1978) illustrates this possibility and argues that exogeneity, in the sense of being determined outside the system, cannot be tested using nonexperimental data.

concepts of causality. Wold (1954) advocates the notion of *causal chains* between variables in order to specify a recursive structure for a system of simultaneous equations. However, Basmann (1965) shows that it is impossible to identify a unique direction of causality when the relationship between the variables is strictly contemporaneous. In contrast, the Granger concept of causality based on temporal ordering or predictability can be tested by determining whether y_t is related to past, current, or future values of x in addition to past values of y.

In a physical system, the principle of *post hoc ergo propter hoc* can be readily related to "causation." For example, if it rains and then the pond fills up, it is easy to believe that the rain caused the pond to fill up. However, in economic systems it is less clear that temporal ordering and causality should be synonymous. Economic agents make decisions based on expectations of what state of the world will occur in the future, and the process of forming expectations about the future can change the interpretation of Granger causality. The concept of *rational expectations* (Muth, 1961) or *efficient markets* (Fama, 1970) suggests that this problem will occur whenever one deals with a market where arbitrage profits could be made if actual prices deviate from expected prices in a systematic way.

For example, Fama (1975) has analyzed the relationship between the monthly Consumer Price Index (CPI) inflation rate and the nominal return on a one-month treasury bill, which is known at the beginning of the month. He finds that the treasury bill rate predicts the subsequently observed inflation rate. Subsequent work by Nelson and Schwert (1977) indicates that the treasury bill rate *causes* the rate of inflation in the Granger sense, since the treasury bill rate adds significant information beyond that contained in past inflation rates for predicting inflation. However, this interpretation of the relationship between interest rates and inflation is misleading. An alternative interpretation of these empirical results is that the treasury bill rate contains an efficient assessment of the expected inflation rate, so that interest rates adjust to different levels of *expected* inflation over time. In this scenario, predictable movements of inflation *cause* movements in the interest rate in the usual sense of the word.

Thus, the Granger concept of causality based on temporal ordering will not lead to sensible conclusions about directions of causation in many instances. Zellner (1977, 1979) provides a valuable discussion of this and other problems with temporal ordering as a definition of causality. Nevertheless, this definition of causality provides a focus for empirical work designed to determine the relationships between economic time series variables. For example, if there is feedback between y and x, usual regression techniques applied to one-way distributed lag models would often yield inconsistent parameter estimates.[3] Thus, tests of Granger causality can be an important part of the analysis of the specification of econometric models between time series variables.

III. Some Examples

Sims (1972) uses seasonally adjusted quarterly data from 1947-69 to test for unidirectional causality (exogeneity) in the relationships between gross national product (GNP) and the nominal money supply defined in two ways: (*a*) the monetary base (MB), which is currency plus reserves adjusted for changes in reserve requirements, and (*b*) M1, which is currency plus demand deposits. Sims estimates two-sided distributed lag equations:

$$y_t = a + \sum_{i=-4}^{8} \beta_i x_{t-i} + \gamma \underline{z}_t' + \epsilon_t ,$$

where y_t and x_t are transformed values of GNP, MB, or M1 (regressions of both GNP on money, and money on GNP are estimated), and $\underline{z}_t$ represents a vector of seasonal dummy variables and a time trend variable. Sims then uses F-tests to determine the joint significance of the lead coefficients $(\beta_{-4}, \ldots, \beta_{-1})$ and the lag coefficients $(\beta_0, \beta_1, \ldots, \beta_8)$ from each of the regressions. Because Sims recognizes the importance of having serially uncorrelated disturbances in his regression, he uses the natural logarithms of all of his variables, and he uses the autoregressive filter $(1\text{-}0.75L)^2 = (1\text{-}1.5L + 0.5625L^2)$ to transform each of the variables. For example,

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[3]If $\{x_t\}$ is autocorrelated and the true relationship between y and x is a two-sided distributed lag,

$$y_t = a + \sum_{i=-k}^{m} \beta_i x_{t-i} + \epsilon_t ,$$

then least squares estimators of the regression coefficients for the one-sided distributed lag,

$$y_t = a + \sum_{i=0}^{m} \beta_i x_{t-i} + \eta_t ,$$

are biased and inconsistent because the disturbance, $\eta_t = \sum_{i=-k}^{-1} \beta_i x_{t-i} + \epsilon_t$, is correlated with the regressors.

$$y_t \equiv \ln GNP_t - 1.5 \ln GNP_{t-1} + 0.5625 \ln GNP_{t-2}$$

is the transformed value of GNP.

On the basis of F-tests, Sims (1972) concludes that although GNP is not exogenous to money, there is "no evidence that appears to contradict the common assumption that money can be treated as exogenous in a regression of GNP on current and past money" (p. 550). However, as a result of several types of tests on the regression residuals, Sims notes that "there is room for doubt about the accuracy of the F-tests on regression coefficients" (p. 549).

Indeed, Feige and Pearce (1974) reexamine Sims's tests using different types of prefilters and note that Sims's results do not hold up under some choices of transformations of the variables. In particular, when they analyze estimates of the innovations of money and income, which are the residuals from univariate ARIMA models for each of the variables, they cannot reject the hypothesis that there is no relationship between money and income at usual significance levels.

Williams, Goodhart, and Gowland (1976) use similar data from the United Kingdom for the 1958-71 period to test for causal relationships between money and income. Using different transformations, including first differencing, they find no strong noncontemporaneous relationships between money and income in the U.K. In fact, the regression relationships between transformed money and transformed income are so weak that Williams, Goodhart, and Gowland cannot reject the hypothesis that all of the regression coefficients are zero at usual significance levels. Thus, they cannot reject the null hypothesis that nominal income is unrelated to the money supply.

Rutner (1975) analyzes the relationships between the monetary base and M1 using spectral analysis and finds little relationship between these time series after they have been transformed or filtered. He estimates a high order autoregression for each variable and then takes the residuals from that regression as his "detrended" series. Rutner finds no significant contemporaneous relationship between the transformed money supply (M1 or M2) and the transformed monetary base (adjusted or unadjusted for changes in reserve requirements). However, spectral analysis of the transformed series indicates a statistically significant long-run (low frequency) relationship between the transformed series, with some indication that the base leads the money supply in the long run.

Feige and Pearce (1976) examine the relationship between several definitions of the money supply (M1, M2, or MB) and the price level as measured by the Consumer Price Index (CPI) or the Wholesale Price Index (WPI) during the

1953-71 period. They estimate ARIMA models for monthly and quarterly versions of the variables and use the residuals from these models as estimates of the innovations for the respective series. They perform two tests to determine causal relationships based on the cross-correlations between the residuals from the monetary variables and the residuals from the price variables. First, they compute the correlation coefficients between the residuals from a price series, a_{yt}, and the residuals from a monetary series, a_{xt},

$$r_{a_y a_x}(i) = \text{corr}\,(a_{yt}, a_{xt-i})\,.$$

They compare each individual cross-correlation estimate with its asymptotic standard error (which is $T^{-1/2}$ under the null hypothesis of zero cross-correlations), and they look for estimates which are more than two standard errors different from zero for $i = 12, \ldots, 0, \ldots, -12$. Second, they use a joint test developed by Haugh (1972) to test whether all of the cross-correlations are zero.[4] On the basis of these test procedures, Feige and Pearce "could not reject the hypothesis that the rate of inflation is causally independent of the monetary aggregates. . .which appears to be in direct conflict with both popular doctrine and a substantial body of published econometric literature" (p. 519).

Pierce (1977a) examines the causal relationships between a variety of economic time series variables using weekly data from September, 1968 through April, 1974. For example, using tests based on Haugh's S-statistic, he cannot reject the hypothesis that demand deposits (DD) are unrelated to the 90-day treasury bill rate (TB) at all leads and lags.

These examples, which highlight existing applications of time series techniques to questions of causal relationships between economic variables, indicate some of the puzzling and conflicting results which have been derived using different methodologies. Since the procedure of analyzing the innovations of different series has surprisingly failed to detect any substantial relationship between economic variables such as the money supply and nominal income or the price level, the remainder of the paper investigates the properties of the new time series methodologies.

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[4]The statistic

$$S = T \sum_{i=-M}^{M} [r_{a_y a_x}(i)]^2$$

has an asymptotic χ^2 distribution with $2M + 1$ degrees of freedom under the null hypothesis that all $2M + 1$ cross-correlations are zero. Note that S can be defined over any range of the cross-correlation function; it does not have to be symmetric around lag zero. Appendix B discusses these test procedures further.

IV. Time Series Methods for Analyzing Causality

On the basis of work by Haugh (1972, 1976) and Haugh and Box (1977), Pierce and Haugh (1977) suggest a two-step procedure for implementing tests of causality. First, each variable is transformed to have a constant unconditional mean and variance over the sample period (possibly by using logarithmic and/or differencing transformations of the raw data). For example, the rates of change of many economic time series, the first differences of the natural logarithms, are stationary in this sense. Then, univariate autoregressive-moving average (ARMA) models are estimated for the transformed variables[5]

$$\phi_y(L)y_t = a_1{}' + \theta_y(L)a_{yt},$$

$$\phi_x(L)x_t = a_2{}' + \theta_x(L)a_{xt}, \tag{2}$$

where $\phi_y(L)$ and $\phi_x(L)$ are finite autoregressive polynomials in the lag operator; $\theta_y(L)$ and $\theta_x(L)$ are finite moving average polynomials in the lag operator; and a_{yt} and a_{xt} are each serially uncorrelated. Based on the univariate models for y and x in (2), the unexpected part of y which could not be predicted on the basis of its past history is a_{yt}. Similarly, a_{xt} is the part of x_t which could not be predicted on the basis of its past history. The disturbances a_{yt} and a_{xt} are referred to in the time series literature as the "innovations" of the ARMA processes in (2).

Note that the current value of a stationary ARMA process can always be represented as a weighted sum of the current and past innovations

$$y_t = \sum_{i=0}^{\infty} \Psi_i a_{yt-i}, \qquad \Psi_0 = 1,$$

where the Ψ_i weights are functions of the autoregressive and moving average parameters. The "systematic" or predictable part of the ARMA process is also a weighted sum of the past innovations

[5] Box and Jenkins (1976) and Nelson (1973) discuss procedures for specifying and estimating univariate ARMA models.

$$\hat{y}_t = y_t - a_{yt} = \sum_{i=1}^{\infty} \Psi_i a_{yt-i} .$$

Thus, analyzing the innovations of the time series does not eliminate or throw away the systematic part of the variable.

The second step in the Pierce-Haugh causality test is to examine the cross-correlations between a_{yt} and past, current, and future values of a_x, which is referred to as the cross-correlation function between a_y and a_x (as a function of lag i). Pierce and Haugh (1977) prove that the innovations, a_y and a_x, yield conclusions about causality identical to those about the transformed variables, y and x, in (2). For example, if "a_x causes a_y (but not instantaneously)" and "a_y does not cause a_x," then "x causes y (but not instantaneously)" and "y does not cause x." Nevertheless, the distributed lag model between the innovations series, $\{a_{yt}\}$ and $\{a_{xt}\}$, can be substantially different from the distributed lag model between the original variables, $\{y_t\}$ and $\{x_t\}$.

For example, consider the distributed lag model in (1) where y does not cause x, but x causes y,

$$y_t = a + \beta(L)x_t + \eta_t ,$$

where $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \ldots$

is a polynomial in the lag operator, and η_t is a stationary disturbance which may be autocorrelated. Assume that η_t follows an ARMA process,

$$\phi_N(L)\eta_t = \theta_N(L)\epsilon_t ,$$

where $\{\epsilon_t\}$ is "white noise," serially independent, identically distributed variables with mean zero and constant variance, σ_ϵ^2. The univariate ARMA representation of x_t in (2) can be substituted into (1) to yield

$$y_t = a' + \beta(L) \frac{\theta_x(L)}{\phi_x(L)} a_{xt} + \frac{\theta_N(L)}{\phi_N(L)} \epsilon_t .$$

Further substitution of the ARMA representation of y_t from (2) yields

$$\frac{\theta_y(L)}{\phi_y(L)} a_{yt} = \beta(L) \frac{\theta_x(L)}{\phi_x(L)} a_{xt} + \frac{\theta_N(L)}{\phi_N(L)} \epsilon_t$$

or

$$a_{yt} = \beta(L) \frac{\phi_y(L)}{\theta_y(L)} \frac{\theta_x(L)}{\phi_x(L)} a_{xt} + \frac{\phi_y(L)}{\theta_y(L)} \frac{\theta_N(L)}{\phi_N(L)} \epsilon_t , \qquad (3)$$

which is simply a distributed lag model for a_{yt} in terms of current and lagged values of a_x with ARMA disturbances,

$$a_{yt} = \nu(L) a_{xt} + \omega(L) \epsilon_t . \qquad (4)$$

Thus, if there is a one-sided distributed lag model between y and x such as (1), there is a corresponding one-sided distributed lag model between the innovations for the univariate ARMA models, a_y and a_x. We can test for "causal" relationships between time series variables using either the original variables or the innovations.

However, the distributed lag model between the innovations in (3) can be substantially different from the distributed lag model between the original variables in (1). The coefficients between y and x,$\beta(L)$, are generally different from the coefficients between a_y and a_x, $\nu(L)$. The contemporaneous coefficient is the same ($\beta_0 = \nu_0$), but it is necessary to consider the form of the ARMA models for y and x in order to determine the relationship between the lagged coefficients in $\beta(L)$ and $\nu(L)$.

The autocorrelation properties of the disturbances in (3) are also generally different from the autocorrelations of the disturbances of the original model (1). The ARMA models for y and x make a_y and a_x serially uncorrelated. Given the usual assumption that a_x and ϵ are independent, the distributed lag polynomial, $\nu(L) = \beta(L) \dfrac{\phi_y(L)}{\theta_y(L)} \dfrac{\theta_x(L)}{\phi_x(L)}$, places restrictions on the ARMA model for the disturbances in (4), $\omega(L) = \dfrac{\phi_y(L)}{\theta_y(L)} \dfrac{\theta_N(L)}{\phi_N(L)}$. This is easy to see since a_{yt} is just the sum of two independent ARMA processes which must cancel each other to produce a serially uncorrelated series. For example, suppose that ν_0 and ν_1 are nonzero, but all other ν_i are equal to zero,

$$a_{yt} = \nu_0 a_{xt} + \nu_1 a_{xt-1} + N_t .$$

Since a_{yt} and a_{xt} are serially uncorrelated by construction, N_t must follow a first order moving average process

$$N_t = \epsilon_t - \theta_1 \epsilon_{t-1} .$$

The magnitude of θ_1 depends on the ratio ν_1/ν_0 and the relative variances of a_{xt} and ϵ_t.[6]

A few special cases should illustrate the differences between the distributed lag model for the original variables in (1) and the distributed lag model for the innovations in (3).

> *Case 1*: Suppose that y and x have exactly the same ARMA representations, $\phi_y(L) = \phi_x(L)$ and $\theta_y(L) = \theta_x(L)$. In this special case, the distributed lag model between the innovations is exactly the same as the distributed lag model between the original variables,

$$\nu(L) = \beta(L) \frac{\phi_y(L)}{\theta_y(L)} \frac{\theta_x(L)}{\phi_x(L)} = \beta(L) .$$

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[6] **Box and Jenkins (1976, pp. 121-25) and Haugh and Box (1977, p. 126) discuss the relationships between $\nu(L)$ and $\omega(L)$ which are implied by the fact that a_{yt}, a_{xt}, and ϵ_t are all serially uncorrelated in (4).**

However, the autocorrelation structure of the disturbances of the innovations model,

$$\frac{\phi_y(L)}{\theta_y(L)} \frac{\theta_N(L)}{\phi_N(L)} \epsilon_t ,$$

is generally different from the autocorrelation structure of the disturbances of the original model.

Case 2: Suppose there is a strictly *contemporaneous* relationship between y_t and x_t in (1), $\beta_0 \neq 0$ and $\beta_i = 0$ for $i = 1, 2, \ldots$. In general, the distributed lag coefficients between the innovations are nonzero at all lags, unless the ARMA models for y and x are identical. For example, suppose that y_t follows the first-order AR process with $\phi_y(L) = (1 - 0.9L)$ and $\theta_y(L) = 1$, and x_t is serially uncorrelated, so $\phi_x(L) = \theta_x(L) = 1$. Then

$$\nu(L) = \beta_0 \phi_y(L) . \tag{5}$$

By matching coefficients on both sides of equation (5), we can solve for the coefficients of $\nu(L)$:

$$\nu_0 = \beta_0 ,$$

$$\nu_1 = -0.9\beta_0 ,$$

$$\nu_j = 0 , \qquad j > 1 .$$

Note that if either $\theta_y(L)$ or $\phi_x(L)$ had been a polynomial of order greater than zero, the coefficients of $\nu(L)$ would generally be nonzero at all lags. Also, note that the *steady-state gain* (total multiplier)[7] between x and y is $\sum_{i=0}^{\infty} \beta_i = \beta_0$, while the gain between a_x and a_y is only one tenth as large, $\sum_{i=0}^{\infty} \nu_i = \beta_0 - 0.9\beta_0 = 0.1\beta_0$.

In this case, the relationship between y_t and x_t is only contemporaneous, so there is no easy way to identify a unique direction of causality based on equation (1). Nevertheless, there is evidence that x causes y from the relationship between a_y and a_x, since ν_1 is nonzero.

The disturbance term in the model for the original variables must follow an ARMA process, since y is serially correlated, while x is not. The disturbance in the model for the innovations must follow a first order moving average process.

Case 3: Suppose there is a Koyck distributed lag relationship between y and x,

$$y_t = a + \frac{\beta_0}{1 - \delta L} x_t + \eta_t ,$$

where $\frac{\beta_0}{1 - \delta L} = \beta_0(1+\delta L+\delta^2 L^2+\delta^3 L^3+. . .)$, so the distributed lag coefficients between x and y decay at the geometric rate: $\beta_j = \beta_0 \delta^j$. For example, suppose $\delta = 0.9$ and $\beta_0 = 0.1$, so that the steady-state gain between y and x is 1.0. As in Case 2, suppose that y_t follows the first order AR process with $\phi_y(L) = (1 - 0.9L)$ and $\theta_y(L) = 1$, and x_t is serially uncorrelated, so $\phi_x(L) = \theta_x(L) = 1$. In this case, the relationship between a_y and a_x is strictly contemporaneous:

$$\nu(L) = \beta(L)\phi_y(L) = \frac{\beta_0}{(1 - 0.9L)} (1 - 0.9L) = \beta_0 .$$

[7] The steady-state gain can be thought of as the long-run change in the level of y if x is set equal to one in all future periods. It represents the cumulative effect on all future values of y of the current value of x_t.

> Thus, even though it appears that x causes y based on the one-sided distributed lag model between y and x, the relationship between a_y and a_x is strictly contemporaneous, so there is only instantaneous causality between y and x in this case. As in Case 2, the gain between a_x and a_y is only one-tenth as large as the gain between x and y.

These three cases highlight the differences between the distributed lag models for the original variables in (1) and for the innovations in (3). Case 1, where the ARMA models for y and x are identical, represents the only case where the distributed lag coefficients are the same at all lags for the two models. The last two cases illustrate the difficulty of determining anything about *simple causality* from the distributed lag model for the original variables. In Case 2, there is only a contemporaneous relationship between y and x, but there is a lagged causal relationship between the innovations, current a_y and past a_x. On the other hand, Case 3 involves a distributed lag of y on x, but only a contemporaneous relationship between a_y and a_x, so there is no evidence of simple causality (since lagged values of x or a_x *alone* cannot reduce the variance of the error in predicting y_t).[8]

Since the innovations $\{a_{x\,t}\}$ are serially uncorrelated, the distributed lag coefficients between a_y and a_x are proportional to the cross-correlation coefficients between $a_{y\,t}$ and $a_{x\,t-i}$. These illustrations indicate that the size and pattern of the cross-correlations between the innovations should be considered in relation to the implied values of the distributed lag coefficients for the relationship between the original variables. In the next section, two examples are provided to illustrate the importance of analyzing the implied coefficients of $\beta(L)$ in (1) as an integral part of the analysis of the innovations series.

V. Lack of Relationships?

A. Inflation and the Money Supply

As mentioned in Section III, Feige and Pearce (1976) analyze the relationships between monetary growth rates and inflation using the time series techniques described in the previous section and cannot reject the hypothesis that these variables are unrelated. Such a finding, if true, could have profound implications for the study of monetary economics. However, the inability to reject the null hypothesis does not confirm the hypothesis that the inflation

[8]Pierce (1975, pp. 355-56); Nelson (1975a, p. 342); and Pierce and Haugh (1977, pp. 274-75) note that this result occurs whenever $\beta(L)\cdot x_t$ has the same stochastic structure as the disturbance η_t in (1).

rate is unrelated to the growth rate of the money supply. It is necessary to consider the *power* of the test, the probability that the null hypothesis is rejected when it is false, before concluding that there is no relationship between inflation and monetary growth.

In order to analyze the Feige-Pearce results, monthly data from July, 1953-June, 1971 are used to estimate ARIMA models for the CPI inflation rate (not seasonally adjusted),

$$(1-L^4)(1-L)\rho_t = \underset{(1.80 \times 10^{-6})}{0.30 \times 10^{-6}} + (1-\underset{(0.01)}{0.94}L^4)(1-\underset{(0.03)}{0.88}L)a_{yt} \tag{6}$$

$$S(a_y) = 0.00194 \qquad Q^{a_y}(10) = 13.1 ,$$

and for the rate of growth of the monetary base (not seasonally adjusted or adjusted for changes in reserve requirements),

$$(1-L^{12})m_t = \underset{(0.07 \times 10^{-3})}{0.37 \times 10^{-3}} + (1-\underset{(0.02)}{0.89}L^{12})a_{xt} \tag{7}$$

$$S(a_x) = 0.00448 \qquad Q^{a_x}(11) = 16.7 ,$$

where standard errors are in parentheses, $S(a)$ is the standard deviation of the residuals, and $Q^a(K)$ is the Box-Pierce (1970) statistic for 12 lags of the residual autocorrelation function which has a χ^2_K distribution in large samples under the hypothesis that all residual autocorrelations are zero. These ARIMA models are of the same form as the models used by Feige and Pearce, although the parameter estimates are somewhat different.[9] For the purposes of this illustration, the important thing to note is that the residuals, a_y and a_x, are not serially correlated.

Table 1 presents estimates of the cross-correlations between the price residuals, a_{yt}, and the monetary base residuals, $a_{x\,t-i}$, for $i = -12, \ldots, 0, \ldots, 12$. None of the estimated cross-correlations is more than two standard errors from zero, and there is no obvious pattern in the cross-correlation function.

[9] The differences may be attributable to several things. The estimates in (6) and (7) are obtained from an unconditional maximum likelihood procedure (see Box and Jenkins, 1976, pp. 212-20), whereas the Feige-Pearce estimates may be conditional on the initial conditions of the ARIMA process (see Box and Jenkins, 1976, pp. 209-12). Also, Feige and Pearce use monetary base data from the *Federal Reserve Bulletin*, while the data used in this paper are from the Federal Reserve Bank of St. Louis.

Table 2 presents Haugh's S-statistic for various combinations of leads and lags of the cross-correlation function.[10] None of these test statistics would reject the hypothesis that the monetary base and the CPI are unrelated at usual significance levels. In fact, all of the S-statistics are near their mean values under the null hypothesis of no relationship.

Although the cross-correlations between residuals in Table 1 are somewhat different from the estimates plotted by Feige and Pearce (1976, p. 513), the test results in Tables 1 and 2 lead to the same disturbing conclusion reached by Feige and Pearce: the CPI inflation rate, ρ, seems to be unrelated to the growth rate of the monetary base, m. However, the distributed lag model between ρ and m which is implied by the cross-correlations of the residuals in Table 1 gives a different impression of the relationship between the monetary base and the CPI.

Column (2) of Table 3 presents estimates of the distributed lag coefficients, ν_i, between the innovations, a_{yt} and $a_{x\,t-i}$, which are proportionately smaller than the cross-correlations in Table 1. Column (3) of Table 3 contains the coefficients of the distributed lag model between ρ and m,

$$\rho_t = \sum_{i=0}^{12} \beta_i m_{t-i} + \eta_t ,$$

which are implied by the ARIMA models for ρ and m and the estimates of ν_i in column (2). The footnotes to Table 3 and Appendix C describe the details of the calculations.

Taken at face value, the numbers in column (3) say that a 1 percent increase in the growth rate of the monetary base has a negligible effect on the current inflation rate, but the current growth rate of the monetary base increases the inflation rate in succeeding months by about 0.04 percent per month. Thus, a 1 percent increase in m leads to a 0.52 percent increase in ρ after one year. Such a finding is quite consistent with accepted beliefs about the time lag between a change in the growth rate of the money supply and a subsequent change in the inflation rate. However, when one realizes that most of the coefficients in column (2) have standard errors which are larger than the estimates, it is apparent that the set of implied values of β_i in column (3) is fortuitously in conformance with previous findings. In fact, a very wide range of patterns of β_i is consistent with the cross-correlations of the innovations in Table 1. Thus, even if knowledge of past monetary growth rates does not

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[10] **Note that the asymptotic distribution of S is χ^2 only in the case where *all* cross-correlations are zero, whether they are included in the computation of S or not. Pierce (1977a, p. 15), Sims (1977b, p. 24), Pierce (1977b, p. 25), and Pierce and Haugh (1977, p. 284) all discuss this problem.**

TABLE 1

Cross-Correlations between the Residuals for the Consumer Price Index and the Monetary Base

Lag, i	$Corr(a_{yt}, a_{xt-i})$	Lag, i	$Corr(a_{yt}, a_{xt-i})$
12	0.06	-1	0.07
11	0.06	-2	-0.11
10	0.02	-3	0.00
9	0.03	-4	0.01
8	0.00	-5	0.13
7	-0.03	-6	-0.08
6	-0.01	-7	-0.03
5	0.11	-8	-0.11
4	0.13	-9	0.03
3	0.08	-10	0.07
2	0.07	-11	-0.07
1	0.09	-12	0.03
0	0.00		

Note: Based on data from July, 1953 to June, 1971. The large sample standard error for each estimate is 0.07, under the null hypothesis that the series are unrelated.

TABLE 2

Tests of the Lack of Relationship between Monetary Growth and the Inflation Rate

$$S = T \cdot \sum_{i=L}^{M} [r_{a_y a_x}(i)]^2$$

	L	M	Degrees of Freedom	S
Feedback				
	-24	24	49	46.7
	-12	12	25	26.4
x causes y				
	1	24	24	27.5
	1	12	12	13.9
y causes x				
	-24	-1	24	19.2
	-12	-1	12	12.6

Note: Tests based on the cross-correlations between the residuals from the ARIMA model for the growth rate of the monetary base, a_x, and the residuals from the ARIMA model for the inflation rate of the CPI, a_y.

TABLE 3

Distributed Lag Model between the CPI and the Monetary Base Implied by Cross-Correlations of the Innovations

Lag i (1)	Estimates of Distributed* Lag Coefficients for Innovations, $\hat{\nu}_i$ (2)	Implied Distributed† Lag Coefficients for Original Variables, β_i (3)	Cumulative Sum of β_i, $\sum_{k=0}^{i} \beta_k$ (4)
0	0.001	0.001	0.001
1	0.041	0.041	0.041
2	0.028	0.033	0.074
3	0.033	0.041	0.115
4	0.058	0.070	0.186
5	0.047	0.069	0.255
6	-0.006	0.022	0.277
7	-0.012	0.015	0.292
8	0.001	0.029	0.320
9	0.011	0.041	0.362
10	0.009	0.037	0.399
11	0.026	0.056	0.455
12	0.026	0.060	0.515

*$\hat{\nu}_i = r_{a_y a_x}(i) \cdot S(a_y)/S(a_x)$, where $S(a_y)$ and $S(a_x)$ are the estimates of the standard deviations of a_y and a_x from equations (6) and (7).

†Implied values of β_i are computed from the estimates of ν_i using the relationship

$$\beta(L) = \nu(L) \; \frac{\theta_y(L)}{\phi_y(L)} \; \frac{\phi_x(L)}{\theta_x(L)} \; .$$

Based on the time series models for the CPI and the monetary base in equations (6) and (7),

$$\beta(L) = \nu(L) \left\{ \frac{(1 - 0.88L)(1 - 0.94L^4)}{(1 - L)\,(1 - L^4)} \right\} \left\{ \frac{(1 - L^{12})}{(1 - 0.89L^{12})} \right\} .$$

The β_i coefficients can be obtained by matching coefficients of the polynomials on each side of the equation. Appendix C contains some representative calculations.

provide a substantial improvement in predictions of future inflation rates, it is not appropriate to assume that the series are literally unrelated. Perhaps more conventional regression techniques applied to the analysis of the original data, ρ and m, can provide more powerful tests of specific hypotheses of interest to monetary economists.[11]

B. Demand Deposits and the Treasury Bill Rate

As mentioned in Section III, Pierce (1977a) cannot reject the hypothesis that demand deposits, DD, are unrelated to the 90-day treasury bill rate, TB, at all leads and lags using the time series techniques described in Section IV. Pierce does not report the cross-correlations between the innovations of these series, but he does report the ARIMA models used to construct the innovations series. Table 4 presents three hypothetical sets of coefficients relating demand deposits to current and lagged values of the treasury bill rate on a weekly basis,

$$\mathrm{DD}_t = a + \sum_{i=0}^{9} \beta_i \mathrm{TB}_{t-i} + \eta_t ,$$

along with the coefficients of the distributed lag models between the innovations of DD and TB,

$$a_{yt} = \sum_{i=0}^{9} v_i a_{x\,t-i} + N_t , \tag{8}$$

which are implied by the ARIMA models Pierce reports. Even when the relationship between DD and TB is strictly contemporaneous in column (1) of Table 4, the distributed lag coefficients between the innovations are small, erratic, and spread over time. The cumulative effects through ten lags for the innovations are less than half of the steady-state gain between TB and DD. The steady-state gain between a_x and a_y is only one-third of the gain between the original variables.[12]

[11] Plosser (1976, pp. 106-11) uses similar data, including the growth rate of industrial production as an additional regressor, and finds a more significant distributed lag model between ρ and m which has coefficients similar to those in column (3) of Table 3.

[12] The steady-state gain between a_x and a_y can be determined by evaluating the formula

$$\nu(L) = \beta(L)\ \frac{\phi_y(L)}{\theta_y(L)}\ \frac{\theta_x(L)}{\phi_x(L)}$$

with $L = 1$.

TABLE 4

Distributed Lag Coefficients between Demand Deposits and the Treasury Bill Rate

Lag i	β_i^* (1)	$\nu_i^\dagger$ (1)	β_i^* (2)	$\nu_i^\dagger$ (2)	β_i^* (3)	$\nu_i^\dagger$ (3)
0	1.0	1.0	0.50	0.50	0.10	0.10
1	0.0	- 0.53	0.30	0.04	0.10	0.05
2	0.0	0.12	0.20	0.10	0.10	0.06
3	0.0	- 0.21	0.0	- 0.17	0.10	0.04
4	0.0	- 0.04	0.0	- 0.06	0.10	0.03
5	0.0	- 0.34	0.0	- 0.22	0.10	0.00
6	0.0	0.51	0.0	0.15	0.10	0.05
7	0.0	- 0.18	0.0	- 0.01	0.10	0.03
8	0.0	0.19	0.0	0.14	0.10	0.05
9	0.0	- 0.09	0.0	- 0.02	0.10	0.04
Sum of the Coefficients	1.0	0.44	1.0	0.45	1.0	0.46

Note: Pierce (1977a) uses weekly data from September, 1968 through April, 1974 to estimate the ARIMA model,

$$(1 - L)\,TB_t = (1 - 0.30L - 0.10L^5 + 0.12L^6)a_{xt}$$

for the 90-day treasury bill rate, and the ARIMA model,

$$(1 - L)\,DD_t = (1 + 0.23L + 0.18L^3 + 0.18L^4 + 0.32L^5 - 0.13L^6 + 0.16L^{16} - 0.32L^{22} + 0.17L^{34} + 0.16L^{38} + 0.18L^{45})a_{yt}$$

for demand deposits. A periodic seasonal mean is subtracted out of each variable prior to estimating the ARIMA models.

*Coefficients for the hypothethical distributed lag model for the original variables,

$$DD_t = \alpha + \sum_{i=0}^{9} \beta_i TB_{t-i} + \eta_t .$$

†Coefficients for the distributed lag model for the innovations,

$$a_{yt} = \sum_{i=0}^{9} \nu_i a_{xt-i} + N_t ,$$

implied by the ARIMA models for TB_t and DD_t and the assumed values of $\beta_0, \beta_1, \ldots, \beta_9$.

Columns (2) and (3) in Table 4 show distributed lag models between DD and TB which are spread over 3 and 10 weeks with the same total impact. It is not difficult to believe that a change in the treasury bill rate would lead to changes in demand deposits for several subsequent weeks, so columns (2) and (3) probably present a more realistic set of assumptions about $\beta(L)$ than column (1). The implied coefficients of (8), the distributed lag model for the innovations, contained in columns (2) and (3) are small and erratic with no discernible pattern, and the cross-correlations of the innovations would be proportional to the ν_i coefficients.

These calculations are presented to illustrate the difficulty of detecting relationships between DD and TB based on the cross-correlations between the innovations of these series. As with the Feige and Pearce (1976) example, the illustrative calculations in Table 4 show that quite reasonable relationships between the original variables can be difficult to detect from the cross-correlations of the innovations. Thus, failure to reject the hypothesis that the innovations series are unrelated at conventional significance levels should not be the end of the analysis. The power of such procedures against plausible alternative hypotheses should also be investigated.

VI. An Application of Alternative Test Procedures: Interest Rates and Inflation

As a final illustration of the weakness of the Pierce-Haugh tests against specific economic hypotheses, the time series techniques of Section IV are applied to Fama's (1975) model of short-term interest rates as predictors of inflation. Irving Fisher (1930) noted that the nominal interest rate, R_t, can always be viewed as the sum of the expected inflation rate, $E(\rho_t)$, and the expected real rate of interest, $E(r_t)$. The nominal interest rate on a default-free bond is known at the beginning of the period, but the inflation rate and real interest rate are not realized until the end of the period. Fama hypothesizes that the expected real interest rate on short-term U.S. treasury bills was constant over the 1953-71 period, so the expected inflation rate is the nominal interest rate minus the constant expected real rate,

$$E(\rho_t) = R_t - E(r)\,. \tag{9}$$

Fama tests his model using the regression model

$$\rho_t = a + \beta R_t + \epsilon_t \,, \tag{10}$$

where (9) implies that $\beta = 1$ and $a = -E(r)$ in (10). Using monthly data on one-month treasury bill yields and the CPI inflation rate for the January, 1953-July, 1971 period, $\hat{\beta} = 0.98$ with a standard error of 0.10, and the residuals are not substantially autocorrelated. Therefore, Fama concludes that the data support his model as expressed in (9).

*A. **Pierce-Haugh Tests***

In order to carry out the Pierce-Haugh tests, it is necessary to construct ARIMA models for the inflation rate and the interest rate. Following the procedures of Box and Jenkins (1976), the CPI inflation rate from February, 1954 to July, 1971 is modeled as a multiplicative seasonal ARIMA process,

$$\begin{aligned} (1-L)(1-L^{12})\rho_t &= (1 - \underset{(0.03)}{0.87}L)(1 - \underset{(0.02)}{0.92}L^{12})a_{yt}\,, \end{aligned} \tag{11}$$

$$S(a_y) = 0.0019 \qquad Q^{a_y}(10) = 13.9\,,$$

where standard errors are in parentheses under the estimates of the ordinary and seasonal moving average parameters. The Box-Pierce (1970) statistic for 12 lags of the residual autocorrelation function indicates no model inadequacies.

The ARIMA model for the one-month treasury bill rate is of similar form,

$$(1-L)(1-L^{12})R_t = (1 - \underset{(0.07)}{0.27}L + \underset{(0.07)}{0.20}L^2)(1 - \underset{(0.02)}{0.90}L^{12})a_{xt}\,, \tag{12}$$

$$S(a_x) = 0.0003 \qquad Q^{a_x}(9) = 20.1\,.$$

Although the Box-Pierce statistic for the residual autocorrelations of (12) is large, the large autocorrelations occur at lags which are difficult to believe are

TABLE 5

Relationships between Interest Rates and Inflation, and Their Innovations

Lag i (1)	Cross-Correlations* of Innovations, $r_{a_y a_x}(i)$ (2)	Estimates of Distributed Lag Coefficients for Innovations,† $\hat{v}_i$ (3)	Implied Distributed Lag Coefficients for Original Variables,‡ β_i (4)	Sum of the Coefficients $\sum_{k=0}^{i} \beta_k$ (5)	Cross-Correlations of Innovations Implied by Fama's Model,§ $\beta_0 = 1$ (6)
0	0.05	0.32	0.32	0.32	0.16
1	0.09	0.52	0.33	0.65	0.09
2	0.05	0.31	-0.12	0.53	0.11
3	-0.03	-0.15	-0.52	0.01	0.10
4	0.10	0.61	0.62	0.63	0.09
5	0.10	0.59	0.33	0.96	0.08
6	0.00	0.02	-0.53	0.43	0.07
7	0.11	0.64	0.41	0.84	0.06
8	0.00	-0.01	-0.35	0.49	0.05
9	0.13	0.77	0.60	1.09	0.04
10	0.11	0.65	0.21	1.30	0.04
11	0.00	0.02	-0.61	0.69	0.03
12	0.02	0.14	-0.09	0.60	0.03

*The large sample standard error for each of these estimates is 0.07 under the hypothesis that the series are unrelated.

† $\hat{v}_i = r_{a_y a_x}(i) \dfrac{S(a_y)}{S(a_x)}$, where $S(a_y)$ and $S(a_x)$ are the standard deviations of a_y and a_x from equations (11) and (12), respectively.

‡ β_i coefficients determined by solving the equation

$$\beta(L) = v(L) \left\{ \frac{(1 - 0.87L)(1 - 0.92L^{12})}{(1 - 0.27L + 0.20L^2)(1 - 0.90L^{12})} \right\}$$

using the estimates of v_i in column (3).

§ The cross-correlations of the innovations implied by the ARIMA models in equations (11) and (12) and Fama's model that $\beta_0 = 1$ and $\beta_i = 0$, for $i \neq 0$.

important, so (12) is accepted as an adequate univariate time series model for the interest rate.[13]

Column (2) of Table 5 contains the cross-correlations of a_{yt} with a_{xt-i}, for $i = 0,1, \ldots ,12$, where a_y and a_x are the residuals from (11) and (12) respectively. None of these cross-correlation estimates is more than 2 standard errors different from zero, and Haugh's S-statistic for these 13 lags is 15.3, which is just slightly larger than the expected value of the statistic under the hypothesis that the series are unrelated. Thus, the cross-correlations of the innovations suggest that the inflation rate is unrelated to the interest rate.

Column (4) of Table 5 contains the values of the distributed lag coefficients, β_i, between the inflation rate and current and lagged interest rates. These coefficients are derived from the estimates of ν_i in column (3) and the relationship

$$\beta(L) = \nu(L) \left\{ \frac{(1 - 0.87L)(1 - 0.92L^{12})}{(1 - 0.27L + 0.20L^2)(1 - 0.90L^{12})} \right\}.$$

Although the implied contemporaneous coefficient, β_0, is only 0.32, the cumulative sum of the coefficients in column (5) is close to 1. This implies that a 1 percent increase in the treasury bill rate is associated with a 1 percent increase in the inflation rate within a few months.

As an indication that the cross-correlations of the innovations in column (2) of Table 5 are not strong evidence against Fama's hypothesis, the cross-correlations of the innovations which are implied by Fama's model are listed in column (6). First, the relationship

$$\nu(L) = \beta_0 \frac{(1 - 0.27L + 0.20L^2)(1 - 0.90L^{12})}{(1 - 0.87L)(1 - 0.92L^{12})},$$

where $\beta_0 = 1$, is used to solve for the implied distributed lag coefficients between the innovations, ν_i. Then, the relationship

$$r_{a_y a_x}(i) = \nu_i \frac{S(a_x)}{S(a_y)}$$

[13]The time series models in (11) and (12) are the same ones selected by Plosser (1976) after considering a wide variety of model forms. Alternative models yielded similar results in the Pierce-Haugh tests of relationships between interest rates and inflation.

is used to solve for the implied cross-correlations of the innovations. Because the cross-correlations implied by Fama's model in column (6) are small and close to the actual estimates in column (2), the Pierce-Haugh tests do not provide strong evidence against Fama's hypothesis.

B. Parametric Tests of Causality

As mentioned in Section II, Nelson and Schwert (1977) take a more direct approach to testing whether the interest rate contains predictive information about inflation beyond that contained in past inflation rates. Nelson and Schwert embed the interest rate as an additional variable in the time series model for inflation and find that the interest rate does contain significant incremental information. For example, the model

$$(1-L^{12})(1-L)\rho_t = \underset{(0.33)}{0.67}(1-L^{12})(1-L)R_t + (1-\underset{(0.05)}{0.68}L^{12})(1-\underset{(0.05)}{0.74}L)\epsilon_t \qquad (13)$$

is a generalization of both Fama's model (10) and the ARIMA model for inflation (11). If the coefficient of $(1-L^{12})(1-L)R_t$ is zero, (13) specializes to the ARIMA model, and the interest rate does not "cause" inflation in Granger's sense. On the other hand, if both the ordinary and seasonal moving average parameters are equal to one, (13) specializes to Fama's model where all of the information about inflation contained in past inflation is subsumed by the interest rate.[14] Both extreme cases are rejected by the data at usual significance levels. Thus, the test against the specific alternative hypothesis implied by Fama's model can reject the null hypothesis that the variables are unrelated. The test in (13) is more powerful than the Pierce-Haugh test against Fama's alternative hypothesis. All of the diagnostic checks recommended by Box and Jenkins (1976, ch. 11) indicate that (13) is an adequate representation of the relationship between inflation and interest rates.

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[14] Note that a conventional *t*-test of the hypothesis that a moving average parameter equals one cannot be based on the Student-*t* distribution since such a parameter is on the boundary of the admissible parameter space (i.e., the moving average process is not invertible when the MA parameter equals one). Plosser and Schwert (1977) report sampling experiments which can be used as a basis for this test.

VII. Summary and Conclusions

A. Advantages of the Time Series Techniques

The primary motivation for adopting the Pierce-Haugh method is that application of conventional regression techniques to untransformed economic time series variables can often result in "spurious regressions." If both $\{y_t\}$ and $\{x_t\}$ are serially correlated or nonstationary through time, which is often the case with aggregate economic data, regression equations which are estimated using least squares can often yield spuriously "significant" results because of autocorrelated disturbances. Yule (1926) and Granger and Newbold (1974) illustrate the seriousness of this problem. In order to test causality hypotheses using conventional regression procedures, it is important that the disturbances of the regression equation be serially independent, identically distributed random variables. This is often an inappropriate assumption when dealing with the levels of macroeconomic time series variables. Plosser and Schwert (1978) discuss this argument in detail and provide some examples.

A secondary motivation for using the Pierce-Haugh procedure is that estimates of the regression coefficients v_i are unbiased even if some significant lagged values of a_x are omitted from the estimated regression equation. Unbiased regression coefficients result from the fact that the sequence $\{a_{x\,t}\}$ is serially uncorrelated by construction. On the other hand, omitted lagged values of the original regressor x generally cause the estimates of the regression coefficients β_i to be biased, because x_t is correlated with the omitted lagged values, x_{t-k}. Thus, the problem of specifying the length or form of the distributed lag model is less serious when using the serially uncorrelated innovations.

In summary, the Pierce-Haugh technique is relatively simple to apply in situations where the relationship between two time series variables is not well specified a priori. The Pierce-Haugh methodology reduces the problems of model specification required to perform tests of causality hypotheses. However, the increased flexibility does not come without sacrifices in other dimensions; in particular, the power of the Pierce-Haugh tests is likely to be low, relative to other test procedures, against specific alternative hypotheses.

B. Disadvantages of the Time Series Techniques

Even if one wishes to analyze the relationship between the innovations of two variables, the cross-correlation tests advocated by Pierce and Haugh (1977) may be less accurate than comparable statistics derived from the multiple regression

$$a_{yt} = \sum_{i=0}^{M} v_i a_{xt-i} + N_t . \tag{14}$$

For example, the F-statistic which tests the significance of the regression (14) is proportional to Haugh's S-statistic divided by $(1 - R^2)$, where R^2 is the coefficient of determination from (14). $(M+1)\cdot F$ and S have identical asymptotic χ^2 distributions under the null hypothesis that all of the distributed lag coefficients v_i are zero. However, because $(M+1)\cdot F$ is always greater than or equal to S, the critical regions for these tests are not identical when asymptotic results are used. Thus, the null hypothesis of no relationship will be rejected by the F-test but not by the S-statistic. It is not clear which of these tests has the correct significance level in finite samples. Appendix B derives the algebraic relationship between the cross-correlation tests and regression tests in detail.

Of course, if the distributed lag model between a_y and a_x can be specialized, reducing the number of parameters to be estimated, more powerful tests can be constructed. For example, if $v(L)$ can be modeled as a Koyck distributed lag,

$$v(L) = \frac{v_0}{1 - \delta L} ,$$

it is only necessary to estimate two parameters, v_0 and δ, in order to specify all of the v_i coefficients. Each v_i does not have to be estimated as a separate coefficient in the multiple regression (14), as it does when the Koyck restriction is not imposed on the coefficients.

All of the preceding discussion assumes that the innovations a_{yt} and a_{xt} are directly observable (or, equivalently, that we know the form and parameter values of the ARIMA models for y_t and x_t a priori). In practice, it is necessary to identify (specify) the form of the ARIMA model for each variable based on sample autocorrelations and partial autocorrelations (cf. Box and Jenkins, 1976), and then to estimate the parameters of the ARIMA models using the same sample of data. Thus, the residuals from the ARIMA models, $\hat{a}_{yt}$ and $\hat{a}_{xt}$, are estimates of the unobservable innovations, a_{yt} and a_{xt}. Haugh (1972) proves that $\hat{a}_{yt}$ and $\hat{a}_{xt}$ are consistent estimates of the innovations, so the S-statistic has the same asymptotic distribution under the hypothesis that the series are unrelated when the residuals are used in place of the true innovations. Nevertheless, Pierce (1977a) recognizes that the use of residuals to estimate the coefficients of (14) is analogous to the "errors-in-variables" problem:

> It is well known that measurement error biases estimated regression coefficients toward zero when it occurs in the independent variable and inflates their standard errors when it occurs in the dependent variables. These influences would be expected to exert a like effect on the sample residual cross correlations as well. (p. 20)

Thus, tests of the hypothesis that innovations series are uncorrelated with each other at all leads and lags are more likely to accept the null hypothesis of no correlation in finite samples when residuals are used in place of the unobservable innovations.

If the original variables, y_t and x_t, are measured with error, the measurement errors will generally have a different influence on the estimators of the relationship between the innovations than on the estimators of the relationship between the original variables. Although it is impossible to say a priori, in some plausible cases the least squares estimators of the coefficients of $\nu(L)$ will be biased towards zero more than the least squares estimators of the coefficients of $\beta(L)$ (see Plosser and Schwert, 1978, for some examples). Thus, if the original variables are measured with random errors, causality tests based on the estimated innovations series could fail to detect relationships that would be detected using the untransformed data.

Finally, the relative power of the Pierce-Haugh test in comparison with other tests that two variables are unrelated, such as Sims's (1972) test based on the regression of y_t on past and future x, is unknown in general. It is very important that the regression disturbances are serially uncorrelated for Sims's test to be valid, but Hsiao (1977) argues that Sims's test, as well as some other tests, are likely to be more powerful than the Pierce-Haugh test if this condition is satisfied.

C. Summary

The merits of any statistical procedure must be considered in the context of the model and data which are available. In situations where there is no well-formed model to test, a general procedure which is not highly susceptible to specification errors, such as the Pierce-Haugh methodology, may be the best alternative. On the other hand, if one has a model about the relationship between two variables, such as the rate of inflation and the rate of growth in the money supply, the structure of the model will suggest a more powerful test of his hypothesis.

This paper has described and illustrated the new time series methodology for analyzing relationships between economic variables. Several important facts should be considered before adopting this new methodology to test for "causal" relationships between variables. First, the distributed lag coefficients between the innovations can be very different in pattern and magnitude from the distributed lag coefficients between the original variables, depending on the form of the ARIMA models for the variables. Second, statistical tests based on residuals from estimated ARIMA models may accept the null hypothesis of series independence too frequently. Third, existing statistical tests based on the sample cross-correlations between residual series may have low power against plausible alternative hypotheses, especially when short measurement intervals, such as a week or a month, are used.

Given these qualifications, Feige and Pearce's (1976) inability to reject the hypothesis that the rate of inflation is independent of the rate of growth of monetary aggregates or Pierce's (1977a) inability to reject the hypothesis that demand deposits are unrelated to the treasury bill rate should not be alarming. This paper does suggest that future analyses using the Pierce-Haugh methodology concentrate more on the relationship between the model for the innovations and the model for the original variables.

Finally, the semantic distinction between "causality" and "incremental predictability" should be emphasized. Economists are clearly interested in cause-effect relationships for the purpose of policy formulation; the *effect* on the inflation rate *caused* by a change in the rate of monetary growth is an example. On the other hand, forecasters, such as Pierce (1977a), have a legitimate interest in finding the best predictive model for economic variables. All of the variety of tests of Granger causality are clearly applicable in the latter context, but they may be misleading in the former context, because economic agents make decisions based on expectations about future events. Therefore, in the interests of clarity, future tests of Granger causality ought to be called tests of "incremental predictive content."

Appendix A

Representations of Causality

Suppose that $\{y_t\}$ and $\{x_t\}$ follow a covariance-stationary bivariate linear stochastic process,

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix} + \begin{bmatrix} b_{11}(L) & b_{12}(L) \\ b_{21}(L) & b_{22}(L) \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} , \tag{15}$$

where μ_y and μ_x are the unconditional means of y and x, and $b_{11}(L)$, $b_{12}(L)$, $b_{21}(L)$, and $b_{22}(L)$ are all polynomials in the lag operator L, which is defined such that: $L^k x_t \equiv x_{t-k}$. The disturbances, $\underline{u}_t' = [u_{1t}\ u_{2t}]$, have mean zero, $E(\underline{u}_t) = \underline{0}$, variances equal to one, and contemporaneous covariance equal to zero, $E(\underline{u}_t\underline{u}_t') = \underline{I}$, and they are serially uncorrelated within and between series, $E(\underline{u}_t\underline{u}_{t+s}') = \underline{0}$ for $s \neq 0$.[15] Equation (15) is a moving average (MA) representation of the relationship between y and x. Sims (1972) proved that "x does not cause y" if and only if either $b_{11}(L)$ or $b_{12}(L)$ is zero or $b_{11}(L)$ is proportional to $b_{12}(L)$ in equation (15).

Granger's (1969) proof was expressed in terms of the autoregressive (AR) form of the bivariate process,[16]

$$\begin{bmatrix} c_{11}(L) & c_{12}(L) \\ c_{21}(L) & c_{22}(L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \tag{16}$$

where a_1 and a_2 are constant terms. Granger proved that "x does not cause y" if and only if $c_{12}(L) = 0$ in equation (16).

Finally, Sims (1972) proved that y_t can be expressed as a one-sided distributed lag function of current and past x,

$$y_t = a + \beta(L)x_t + \eta_t ,$$

[15] There are many ways to parameterize a bivariate model such as equation (15). I have adopted Sims's (1972) specification that u_{1t} and u_{2t} are independent "white noise" series with unit variance for convenience. Pierce and Haugh (1977) discuss different parameterizations of this model in detail.

[16] Here I assume that processes have both MA and AR representations. See Box and Jenkins (1976) for a discussion of stationarity and invertibility in the context of univariate processes.

with a disturbance η_t which is uncorrelated with past or future x if and only if "y does not cause x." Note that the disturbance η_t can be serially correlated. All three of these representations of causality are equivalent to the definition that "x causes y" if the variance of y_t conditional on past y and past x is less than the variance of y_t conditional on past y alone:

$$\sigma^2 (y_t | y_{t-1}, \ldots, x_{t-1}, \ldots) < \sigma^2 (y_t | y_{t-1}, \ldots).$$

Thus, Granger causality exists if information about x provides more precise predictions about future movements of y than could be made by knowing just the past history of y.

Appendix B

Relationship Between Cross-Correlation Tests and Regression Tests

Suppose that two white noise series $\{a_{yt}\}$ and $\{a_{xt}\}$ are related through the one-sided distributed lag model,

$$a_{yt} = \sum_{i=0}^{M} \nu_i a_{x\,t-i} + \eta_t \,, \tag{17}$$

where M is a finite integer and η_t is an autoregressive-moving average disturbance.[17] Since $\{a_{xt}\}$ is serially uncorrelated, the least squares estimators of the regression coefficients, ν_i, from the multiple regression (17) are identical to the estimators obtained from the sequence of simple regressions,

$$a_{yt} = \nu_i a_{x\,t-i} + \epsilon_{it} \,, \qquad i = 0,1, \ldots M. \tag{18}$$

The least squares estimator, $\hat{\nu}_i$, is proportional to the estimator of the cross-correlation coefficient between a_{yt} and $a_{x\,t-i}$,

$$r_{a_y a_x}(i) = \hat{\nu}_i \frac{S(a_x)}{S(a_y)} \,, \tag{19}$$

where $S(a_x)/S(a_y)$ is the ratio of the sample standard deviations of a_x and a_y.

Box and Jenkins (1976, pp. 376-77) note that when a_y and a_x are uncorrelated at all lags,

$$\sqrt{T-i} \cdot r_{a_y a_x}(i) \mathrel{\dot{\sim}} N(0,1) \,. \tag{20}$$

In other words, in large samples the cross-correlation coefficient estimator has a Normal distribution with mean of zero and a variance of $1/(T-i)$ when the

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[17] The following analysis applies to the true innovations, but it also generally applies in large samples when residuals from ARIMA models (which are consistent estimators of the true innovations) are used instead.

two series are unrelated. This fact has led Haugh (1972, 1976) and Haugh and Box (1977) to suggest the statistic

$$S = T \cdot \sum_{i=0}^{M} [r_{a_y a_x}(i)]^2, \tag{21}$$

which has a chi-square distribution with $M+1$ degrees of freedom in large samples under the hypothesis that the series are uncorrelated at all lags.

Tests of Individual Coefficients

Note that (20) can be used to test whether any individual cross-correlation coefficient is different from zero by comparing the statistic $\sqrt{T-i}\, r_{a_y a_x}(i)$, with a standard Normal distribution. This is analogous to the t-ratio from the simple regression in (18), and these statistics are related in the following way. The estimator of the sampling variance of $\hat{\nu}_i$ from (18) is

$$S^2(\hat{\nu}_i) = \frac{S^2(\epsilon_i)}{(T-i)S^2(a_x)},$$

where $S^2(\epsilon_i)$ is the estimator of the variance of the disturbance in (18). Thus, the t-ratio from (18) is

$$t = \frac{\hat{\nu}_i}{S(\hat{\nu}_i)} = \frac{\sqrt{T-i}\, S(a_x)\hat{\nu}_i}{S(\epsilon_i)}. \tag{22}$$

Using the relationship in (19),

$$t = [\sqrt{T-i}\, r_{a_y a_x}(i)] \cdot \frac{S(a_y)}{S(\epsilon_i)},$$

where $S(a_y)/S(\epsilon_i)$ measures the degree of association between a_y and $a_{x\,t-i}$. For example, the coefficient of determination for (18) is defined as $R_i^2 = 1 - \frac{S^2(\epsilon_i)}{S^2(a_y)}$, so the t-ratio in (22) is equal to the cross-correlation test statistic divided by $\sqrt{1-R_i^2}$. In large samples, both the cross-correlation test statistic (20) and the regression test statistic (22) have standard Normal distributions under the null hypothesis that $\nu_i = 0$. However, whenever $\hat{\nu}_i$ is not exactly zero, and hence R_i^2 is positive, the regression test statistic will be larger than the cross-correlation test statistic. Therefore, the regression test will reject the null hypothesis more frequently at any level of significance. In other words, even though the cross-correlation test and the regression test have the same large sample distribution, in finite samples they must have different critical regions. The small sample properties of the tests are not well-known.

Note that if more than one of the distributed lag coefficients in (17) is nonzero, the t-ratios obtained from the multiple regression (17) will be larger than the t-ratios obtained from the sequence of simple regressions in (18). This occurs because the variance of the simple regression disturbance ϵ_{it} includes systematic variability due to other lagged values of a_x which are omitted from (18). In terms of the true parameters,

$$\sigma_{\epsilon_i}^2 = \sum_{\substack{j=0 \\ j \neq i}}^{M} \nu_j^2 \sigma_{a_x}^2 + \sigma_\eta^2 ,$$

so $\sigma_{\epsilon_i}^2$ is always larger than the variance of the disturbance from the multiple regression (17), σ_η^2, if more than one value of ν_i is nonzero. Thus, tests of significance on individual lags are more powerful using the multiple regression model (17).

Tests of Sets of Coefficients

If one wants to test the joint hypothesis that a set of coefficients is equal to zero, there is relationship between cross-correlation and regression test statistics which is a direct extension of the previous case, where the tests involve only one coefficient. As mentioned above, the S-statistic defined in (21) has a χ_{M+1}^2 distribution in large samples under the hypothesis that $\nu_i = 0$ for all values of i. An alternative test is based on the F-statistic from the multiple regression (17)

$$F \equiv \frac{\hat{\underline{\nu}}' X' X \hat{\underline{\nu}}}{(M+1) S^2(\eta)} ,$$

which has an F-distribution with $(M+1)$ and $(T\text{-}M\text{-}1)$ degrees of freedom. Note that $\underline{X}'\underline{X}$ is the cross-products matrix, which is just $T \cdot S^2(a_x) \cdot \underline{I}_{(M+1)}$, where $\underline{I}_{(M+1)}$ is an M+1 dimensional identity matrix, since $\{a_{xt}\}$ is serially uncorrelated with constant variance. Thus, it follows that

$$F = \frac{T \cdot S^2(a_x)}{(M+1)S^2(\eta)} \sum_{i=0}^{M} \hat{v}_i^2$$

$$= \frac{S^2(a_y)}{S^2(\eta)} \cdot \frac{1}{(M+1)} \cdot T \cdot \sum_{i=0}^{M} [r_{a_y a_x}(i)]^2$$

$$= \left(\frac{S^2(a_y)}{S^2(\eta)}\right) \cdot \left(\frac{1}{M+1}\right) \cdot S .$$

Therefore, the Haugh S-statistic in (21) is proportional to the multiple regression F-statistic:

$$S = \frac{S^2(\eta)}{S^2(a_y)} \cdot (M+1) \cdot F ,$$

where the ratio $S^2(\eta)/S^2(a_y)$ is equal to one minus the coefficient of determination from (17), $(1\text{-}R^2)$. Because $(M+1) \cdot F$ has a χ^2_{M+1} distribution in large samples, both Haugh's S-statistic and the regression F-statistic have the same large sample distribution; however, the S-statistic is always smaller than the comparable F-statistic when R^2 is nonzero (Hsiao, 1977, p. 17, derives a similar result). Thus, the multiple regression F-test will reject the null hypothesis of no relationship more frequently than tests based on the S-statistic. This does not mean that the F-test is more powerful than Haugh's test; rather, it means that it is not really appropriate to use the same critical region for both tests. The small sample properties of these tests are as yet unknown.

Appendix C

Calculation of Implied Distributed Lag Coefficients

The distributed lag model for the original variables

$$y_t = a + \sum_{i=0}^{\infty} \beta_i x_{t-i} + \eta_t$$

$$= a + \beta(L) x_t + \eta_t \qquad (23)$$

is related to the distributed lag model for the innovations of the original variables

$$a_{yt} = \sum_{i=0}^{\infty} \nu_i a_{x\,t-i} + N_t$$

$$= \nu(L) a_{xt} + N_t \,,$$

as seen in equations (1) and (3) in the text. Specifically, given the ARIMA models for y and x, represented by the autoregressive polynomials, $\phi_y(L)$ and $\phi_x(L)$, and the moving average polynomials, $\theta_y(L)$ and $\theta_x(L)$, the coefficients of $\beta(L)$ can be computed from the coefficients of $\nu(L)$, and vice versa, using the relationship

$$\nu(L) = \beta(L) \frac{\phi_y(L)}{\theta_y(L)} \frac{\theta_x(L)}{\phi_x(L)} .$$

For example, Table 3 contains estimates of ν_i for $i = 0,1, \ldots ,12$, based on the cross-correlations between the innovations of the inflation rate, a_y, and the innovations of the growth rate of the monetary base, a_x. The ARIMA models for these variables imply the following relationship between $\beta(L)$ and $\nu(L)$:

$$\beta(L) = \nu(L) \left\{\frac{(1-0.88L)(1-0.94L^4)}{(1-L)(1-L^4)}\right\} \left\{\frac{(1-L^{12})}{(1-0.89L^{12})}\right\}$$

$$= \nu(L) \left\{\frac{1-0.88L-0.94L^4+0.83L^5-L^{12}+0.88L^{13}+0.94L^{16}-0.83L^{17}}{1-L-L^4+L^5-0.89L^{12}+0.89L^{13}+0.89L^{16}-0.89L^{17}}\right\}. \quad (24)$$

The coefficients of $\beta(L)$ can be obtained by calculating the coefficients of the right side of (24),

$$\beta_0 = \nu_0 ,$$

$$\beta_1 = \beta_0 + \nu_1 - 0.88\nu_0 ,$$

$$\beta_2 = \beta_1 + \nu_2 - 0.88\nu_1 ,$$

$$\beta_3 = \beta_2 + \nu_3 - 0.88\nu_2 ,$$

$$\beta_4 = \beta_3 + \beta_0 + \nu_4 - 0.88\nu_3 - 0.94\nu_0 ,$$

$$\beta_5 = \beta_4 + \beta_1 - \beta_0 + \nu_5 - 0.88\nu_4 - 0.94\nu_1 + 0.83\nu_0 ,$$

$$\beta_6 = \beta_5 + \beta_2 - \beta_1 + \nu_6 - 0.88\nu_5 - 0.94\nu_2 + 0.83\nu_1 ,$$

and so forth. Although this procedure is tedious when the ARIMA models for y and x are complicated, it is necessary to carry out this calculation in order to translate the cross-correlations of the innovations into the coefficients of the original model (23).

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