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EFFECTS OF MODEL SPECIFICATION ON TESTS FOR UNIT ROOTS IN MACROECONOMIC DATA

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Tests for unit roots in autoregressive models (tests for stationarity) are popular in the macroeconomics literature. Monte Carlo experiments in Schwert (1987) show that unit root tests derived for pure autoregressive processes have different sampling distributions when the true process is a mixed autoregressive–integrated moving average (ARIMA) process. Tests suggested by Said and Dickey (1984, 1985), Phillips (1987), Phillips and Perron (1986), and Dickey and Fuller (1979, 1981) are applied to a variety of monthly and quarterly macroeconomic time series to illustrate the effects of ARIMA model specification on inferences about stationarity.

1. Introduction

The question of whether an economic time series is stationary often has important consequences for the interpretation of economic models and data. Following Nelson and Plosser (1982), a number of authors, including Shiller (1981) and Poterba and Summers (1986), have applied the tests proposed by Fuller (1976) and Dickey and Fuller (1979, 1981) to test whether particular time series variables, such as the dividend payments to the Standard & Poor's composite portfolio or the volatility of the Standard & Poor's portfolio, are represented by stationary autoregressive (AR) processes,

$$Y_t = \alpha + \sum_{i=1}^{p} \phi_i Y_{t-i} + u_t, \tag{1}$$

where the roots of the lag polynomial $\phi(L) = (1 - \phi_1 L - \cdots - \phi_p L^p)$ lie outside the unit circle [see Box and Jenkins (1976) for a discussion of stationarity in AR processes]. The null hypothesis in these tests is that the AR process contains one unit root, so that the sum of the autoregressive coeffi-

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cients in (1) equals 1.0. The Dickey–Fuller test involves estimating the model

$$Y_t = \alpha + \rho_\mu Y_{t-1} + \sum_{i=1}^{(p-1)} \phi_i' DY_{t-i} + u_t, \tag{2}$$

where $DY_{t-i} = Y_{t-i} - Y_{t-i-1}$, and the coefficient ρ_μ should equal 1.0 if there is a unit root. Dickey and Fuller use Monte Carlo experiments to tabulate the sampling distribution of the regression 't-statistic', $\tau_\mu = (\hat{\rho}_\mu - 1)/s(\hat{\rho}_\mu)$. The distribution of τ_μ is skewed to the left and has too many large negative values relative to the Student-t distribution. They also tabulate the distribution of the normalized bias of the root estimate $T(\hat{\rho}_\mu - 1)$ for the AR(1) model.[1]

Recently several papers have analyzed the sensitivity of the Dickey–Fuller tests to the assumption that the time series is generated by a pure autoregressive process. Section 2 describes recent extensions of the Dickey–Fuller test procedure suggested by Said and Dickey (1984, 1985), Phillips (1987), Phillips and Perron (1986), and Perron (1986a, b) that attempt to account for mixed ARIMA processes as well as pure AR processes in performing unit root tests. Simulation evidence in Schwert (1987) indicates that when a variable is generated by a mixed ARIMA process the critical values implied by the Dickey–Fuller simulations can be misleading, even for large sample sizes. Section 3 describes several reasons to believe that economic time series are mixed ARIMA processes, rather than pure AR processes. Section 4 performs several unit root tests for a variety of macroeconomic time series, including the monetary base (*MBASE*), bond yields (*BAA*), the Consumer and Producer Price Indexes (*CPI* and *PPI*), nominal wages (*WAGE*), population (*POP*), the labor force (*LAB*), employment (*EMP*), the unemployment rate (*UN*), industrial production (*IP*), the Standard & Poor's composite index of stock prices (*SP500*), the price/earnings ratio for the S&P index (*P/E*), the dividend yield on the S&P index (*D/P*), stock market volatility (*SIG*), the implicit price deflator for *GNP* (*GD*), and nominal and real Gross National Product (*GNP* and *GNP82*). These statistics are compared with the critical values reported in Fuller (1976) and with the relevant critical values from the simulation experiments in Schwert (1987). In many cases, especially for series involving growth rates of nominal magnitudes (e.g., price inflation), inferences about the stationarity of the series depend on the assumed ARIMA model for the data. Use of the unit root tests advocated by Dickey, Fuller, Said, Phillips and Perron can lead to the conclusion that these series are stationary when they may not be. Section 5 discusses the implications of the results for macroeconomic modeling, and section 6 contains brief conclusions.

[1] Dickey and Fuller (1979) show that tests based on the normalized bias are more powerful against the alternative hypothesis that $\rho_\mu < 1$ than tests based on τ_μ.

2. Extensions of the Dickey–Fuller tests

Said and Dickey (1984) argue that an unknown ARIMA$(p, 1, q)$ process can be adequately approximated by an ARIMA$(l, 0, 0)$ process, where $l = O(T^{1/3})$. Given this approximation, the limiting behavior of the unit root test based on a high-order AR approximation will be the same as the Dickey–Fuller results. Of course, for a given application this argument does not indicate the appropriate number of lags l.

Said and Dickey (1985) show that the unit root estimator from an ARIMA$(p, 0, q)$ process,

$$Y_t = \alpha + \rho_\mu Y_{t-1} + \sum_{i=1}^{(p-1)} \phi_i' DY_{t-i} + u_t - \sum_{j=1}^{q} \theta_j u_{t-j}, \tag{3}$$

has the same asymptotic distribution as the Dickey–Fuller estimator. They provide limited Monte Carlo evidence which shows the effect of estimating the moving average parameter θ for an ARIMA(1, 0, 1) model on the unit root test statistic τ_μ.

The distribution of the estimator $\hat{\rho}_\mu$ depends on the structure of the ARIMA process generating the data. As noted by Fuller (1976, pp. 373–382), the statistic $Tc(\hat{\rho}_\mu - 1)$ from a general ARIMA model such as (3) has the same distribution as $T(\hat{\rho}_\mu - 1)$ from the AR(1) model, where the constant c is the sum of the coefficients ψ_i from the moving average representation of the errors from (1) with $p = 1$, $\psi(L) = \theta(L)/\phi(L)$. One strategy for estimating the constant c is to use the coefficients from the ARIMA$(p, 0, 0)$ model in (2) or the ARIMA$(p, 0, q)$ model in (3).[2]

Phillips (1987) shows that the Dickey–Fuller tests are affected by autocorrelation in the errors from (2). He develops modifications of the test statistics τ_μ and $T(\hat{\rho}_\mu - 1)$ that have the asymptotic distributions tabulated by Dickey–Fuller, when the data follow an ARIMA$(p, 0, q)$ process (actually, Phillips allows for more general dependence, including conditional heteroskedasticity). He proposes a two-step procedure, where the first step is to calculate the Dickey–Fuller test statistics from an ARIMA(1, 0, 0) model, and then adjust the Dickey–Fuller statistics using the autocovariances of the errors from (1) with $p = 1$. Phillips modifies the test statistic $T(\hat{\rho}_\mu - 1)$,

$$Z_{\rho\mu} = T(\hat{\rho}_\mu - 1) - 0.5\left(s_{Tl}^2 - s_u^2\right)\left\{T^{-2}\sum_{t=2}^{T}\left(Y_{t-1} - \bar{Y}_{-1}\right)^2\right\}^{-1}, \tag{4}$$

[2] Mark Watson suggested this approach to me. For the ARIMA$(p,0,0)$ model in (2), $c = 1/(1 - \phi_1' - \cdots - \phi_{p-1}')$, where ϕ_i' are the $(p-1)$ autoregressive coefficients for DY_{t-i}. For the ARIMA$(p, 0, q)$ model in (3), $c = (1 - \theta_1 - \cdots - \theta_q)/(1 - \phi_1' - \cdots - \phi_{p-1}')$.

where s_u^2 is the sample variance of u_t,

$$s_{Tl}^2 = T^{-1}\sum_{t=2}^{T} u_t^2 + 2T^{-1}\sum_{j=1}^{l}\omega_{jl}\sum_{t=j+1}^{T} u_t u_{t-j}, \tag{5}$$

and the weights $\omega_{jl} = \{1 - j/(l+1)\}$ ensure that the estimate of the variance s_{Tl}^2 is positive. Following the intuition of Said and Dickey (1984), Phillips suggests that the number of lags l of the residual autocovariances in (5) be allowed to grow with the sample size T. Phillips modifies the regression 't-test' τ_μ,

$$Z_{\tau\mu} = \tau_\mu(s_u/s_{Tl}) - 0.5(s_{Tl}^2 - s_u^2)\left\{s_{Tl}^2 T^{-2}\sum_{t=2}^{T}(Y_{t-1} - \bar{Y}_{-1})^2\right\}^{-1/2}, \tag{6}$$

where s_{Tl}^2 is defined in (5).

Dickey and Fuller also consider tests with a time trend included as an additional regressor, so that the alternative hypothesis is a stationary process around a time trend. Thus, the ARIMA(p,0,0) process in (2) is modified so that

$$Y_t = \alpha + \beta[t - (T+2)/2] + \rho_\tau Y_{t-1} + \sum_{i=1}^{(p-1)} \phi_i' DY_{t-i} + u_t. \tag{7}$$

The ARIMA(p,0,q) model in (3) is modified so that

$$Y_t = \alpha + \beta[t - (T+1)/2] + \rho_\tau Y_{t-1} + \sum_{i=1}^{(p-1)} \phi_i' DY_{t-i} + u_t - \sum_{j=1}^{q} \theta_j u_{t-j}. \tag{8}$$

The regression 't = tests', τ_τ, are important because Evans and Savin (1984) show that τ_μ statistics are a function of the unknown intercept α in (2). Whereas, including a time trend in (7) or (8), even when the trend coefficient $\beta = 0$, makes the distribution of the estimate of the autoregressive parameter $\hat{\rho}_\tau$ independent of α. In empirical applications, where knowledge of the value of the intercept α is unavailable, inclusion of a time trend is probably a prudent decision in performing unit root tests.

Phillips and Perron (1986) develop adjustments to the Dickey–Fuller tests $T(\hat{\rho}_\tau - 1)$ and τ_τ where the alternative hypothesis is a stationary ARIMA(p,0,q) process around a deterministic time trend. As in (4) and (6), the first step in performing these tests is to estimate an ARIMA(1,0,0) model around a time trend [eq. (7) with $p=1$] and then use the residual autocovariances to adjust the Dickey–Fuller statistics. Phillips and Perron show that the test statistic,

$$Z_{\rho\tau} = T(\hat{\rho}_\tau - 1) + \left(s_{Tl}^2 - s_u^2\right)\left(T^6/24D_{XX}\right), \tag{9}$$

has the same asymptotic distribution that Dickey and Fuller tabulate for $T(\hat{\rho}_\tau - 1)$ in the ARIMA(1,0,0) case, where D_{XX} is the determinant of the regressor cross-product matrix. They modify the statistic τ_τ,

$$Z_{\tau\tau} = \tau_\tau(s_u/s_{Tl}) - \left(s_{Tl}^2 - s_u^2\right)T^3\left\{4s_{Tl}\{3D_{XX}\}^{1/2}\right\}^{-1}. \tag{10}$$

This statistic should have the asymptotic distribution tabulated by Dickey and Fuller for τ_τ, even when the regression errors in (7) are autocorrelated.

3. Many economic time series are not pure AR processes

There are many reasons to believe that economic time series contain moving average components. Box and Jenkins (1976, pp. 121–125) show that the sum of two uncorrelated random variables, one of which follows an ARIMA(p, d, q) process and one of which is serially uncorrelated, follows an ARIMA(p, d, Q) process, where $Q = \max\{(p+d), q\}$. Thus, even if an economic variable follows a pure autoregressive process (possibly with d unit roots), if the variable is measured with random error, the measured series will contain a moving average component. For example, if the true variable follows a random walk, ARIMA(0,1,0), but it is measured with independent serially uncorrelated error, the measured series will follow an ARIMA(0,1,1) process.

Muth (1960) discusses the relation between the ARIMA(0,1,1) process and the concepts of permanent and transitory components that have been popular at least since Friedman's (1957) analysis of the permanent income hypothesis. Nelson and Plosser (1982) note that the Muth and Friedman model implies negative autocorrelation at lag one in the first differences of the data.

Time aggregation of data for non-stationary processes also leads to ARIMA(0,1,1) processes for the aggregated data. For example, Working (1960) shows that time aggregated data for a random walk behaves like an ARIMA(0,1,1) process, with an autocorrelation of 0.25 in the first differences of the data. Tiao (1972) analyzes more general nonstationary processes and reaches similar conclusions.

A related phenomenon arises when considering errors from rational forecasts in speculative markets. As discussed by Hansen and Hodrick (1980), multi-period forecast errors of interest rates or exchange rates that are observed every period should have as an ARIMA(0,0,q) process, where q is the length of the forecast horizon minus one. Thus, twelve-month-ahead forecast errors that are observed every month should follow an ARIMA(0, 0, 11) process.

Finally, as stressed by Zellner and Palm (1974), the univariate ARIMA representation of an economic time series variable is implied by the dynamic structure which relates that variable to other economic variables. In quite general circumstances, all variables in a system of simultaneous equations will have a moving average component in their univariate ARIMA representations. For example, Nelson and Schwert (1982) show that a first-order bivariate autoregressive system implies an ARIMA(2, 0, 1) univariate representation for each of the variables.

While there are several econometric reasons to believe that economic time series variables will not be pure autoregressive processes, the most important argument in favor of mixed ARIMA processes is that they fit economic data. Empirical analyses have used mixed ARIMA processes to model the behavior of many important economic variables, including the monthly inflation rate of the U.S. Consumer Price Index [Nelson and Schwert (1977)], the monthly inflation rate of the Israeli Consumer Price Index [Huberman and Schwert (1985)], the logarithm of the monthly standard deviation of the return to the Standard & Poor's composite portfolio [French, Schwert and Stambaugh (1987)], and the quarterly unemployment rate [Nelson (1972)], among others.

4. Unit root tests for macroeconomic data

Table 1 contains a list of the variables that are analyzed in the following tables.[3] There are seven nominal series (*MBASE*, *BAA*, *CPI*, *PPI*, *WAGE*, *GD*, *GNP*), four labor force series (*POP*, *LAB*, *EMP*, *UN*), four stock market series (*SP500*, *P/E*, *D/P*, *SIG*), and two real output series (*IP* and *GNP82*). Ten of the seventeen series are seasonally adjusted. All of the variables are transformed using natural logarithms, except for the series which are expressed as percentage rates already (i.e., *BAA*, *UN*, *P/E*, and *D/P*). Henceforth, all

[3]All of the data, except for the Standard & Poor's composite index and the volatility of the S&P returns, come from the Citibase Databank. The S&P index data represent the value of the index on the last day in the month, rather than the average for the days in the month. The *P/E* and *D/P* series are adjusted similarly. The S&P volatility data are from French, Schwert and Stambaugh (1987); σ_t is an estimate of the standard deviation of the monthly return to the S&P portfolio based on the sum of the squared daily returns within the month, plus twice the sum of the lag 1 cross-products,

$$\sigma_t = \left[\sum_{i=1}^{N_t} r_{it}^2 + 2 \sum_{i=1}^{N_t - 1} r_{it} r_{i+1t} \right]^{1/2},$$

where r_{it} is the rate of return to the S&P portfolio on day i in month t and there are N_t trading days in month t. $SIG_t = \ln \sigma_t$.

Table 1

Macroeconomic time series analyzed in subsequent tables.

Series	Description	Sample period, size
	Monthly data	
MBASE	Log of monetary base, adjusted for changes in reserve requirements (FRB St. Louis, seasonally adjusted)	1/47–12/85 $T = 468$
BAA	Moody's Baa long-term corporate bond yield (not seasonally adjusted)	1/47–12/85 $T = 468$
CPI	Log of consumer price index for urban consumers (*CPI-U*) (seasonally adjusted)	1/47–12/85 $T = 468$
PPI	Log of producer price index for all commodities (not seasonally adjusted)	1/47–12/85 $T = 468$
WAGE	Log of average hourly earnings of production workers in manufacturing (seasonally adjusted)	1/47–12/85 $T = 468$
POP	Log of total civilian non-institutional population (not seasonally adjusted)	1/47–12/85 $T = 468$
LAB	Log of total civilian labor force (seasonally adjusted)	1/48–12/85 $T = 456$
EMP	Log of total employed civilian labor force (seasonally adjusted)	1/48–12/85 $T = 456$
UN	Unemployment rate, all workers 16 years & over (seasonally adjusted)	1/48–12/85 $T = 456$
IP	Log of industrial production (seasonally adjusted)	1/47–12/85 $T = 468$
SP500	Log of Standard & Poor's composite index of stock prices (end-of-month, not seasonally adjusted)	1/47–12/85 $T = 468$
P/E	Price/earnings ratio for Standard & Poor's composite index (not seasonally adjusted)	1/54–12/85 $T = 384$
D/P	Dividend yield for Standard & Poor's composite index (not seasonally adjusted)	1/47–12/85 $T = 468$
SIG	Log of volatility of returns to Standard & Poor's composite index (not seasonally adjusted)	1/47–12/85 $T = 468$
	Quarterly data	
GD	Log of implicit price deflator for gross national product (seasonally adjusted)	1/47–4/85 $T = 156$
GNP	Log of gross national product (seasonally adjusted)	1/47–4/85 $T = 156$
GNP82	Log of real gross national product (1982 dollars, seasonally adjusted)	1/47–4/85 $T = 156$

references to these variables refer to the logs of the variables, where appropriate.[4]

4.1. Autocorrelation of the data

Panel A of table 2 contains the first twelve autocorrelations of these variables, while panel B of table 2 contains the first twelve autocorrelations of the residuals from a regression of the variable against a time trend. As found by Nelson and Plosser (1982) using annual data back to the 19th century, the levels of most of these series are highly autocorrelated. The smallest of the twelve autocorrelation coefficients in panel A is greater than 0.90 for all of the monthly series except the unemployment rate (*UN*) and the four series derived from the S&P composite index (*SP500*, *P/E*, *D/P*, and *SIG*). The autocorrelations for the quarterly data are 0.75 or greater. The detrended series have smaller autocorrelations, although Nelson and Kang (1981) note that this is induced by the detrending procedure, even if the true process contains no trend. The first-order autocorrelations are greater than 0.95 for every series except for stock market volatility (*SIG*).

Note, however, that the question of whether these series follow a stationary process depends on the rate of *decay* of the autocorrelation function. It is the flatness of the autocorrelations, and not just their level, that signifies the presence of an autoregressive root close to or equal to unity. For example, the volatility series has autocorrelations that are virtually equal after the first few lags, which is characteristic of an ARIMA$(0,1,q)$ process, where the q moving average parameters affect the first q autocorrelations, and the remaining lags are similar because of the unit root in the autoregressive part of the model.

Panel A of table 3 contains the first twelve autocorrelations of the first differences of the variables in table 2, and panel B of table 3 contains the first twelve autocorrelations of the residuals from a regression of the first differences against a time trend. The first differences in table 3 are much less autocorrelated than the levels in table 2, although several of the variables have autocorrelations that decay very slowly, including most of the nominal variables (i.e., *MBASE*, *CPI*, *PPI*, *WAGE* and *GD*). Except possibly for the growth rate of the monetary base, detrending the growth rates does not seem to remove the persistence of the autocorrelations in panel B of table 3. These autocorrelations behave much like the autocorrelations of stock market volatility in table 2. Thus, based on the autocorrelations in table 3 it seems as though money growth rates, inflation rates, and population growth rates (*POP*) may be non-stationary, perhaps ARIMA$(0,1,q)$ processes. The economic implica-

[4] Thus, the first differences of the variables measure the growth rates of the raw data, and a time trend for the variables measures an exponential growth path.

Table 2

Sample autocorrelations of the logarithms of monthly and quarterly macroeconomic time series, March 1947 to December 1985 (except where noted).[a]

Series	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	Sample size T
					A. Logarithms of monthly and quarterly data								
MBASE	0.99	0.99	0.98	0.98	0.97	0.96	0.96	0.95	0.94	0.94	0.93	0.92	466
BAA	1.00	0.99	0.98	0.98	0.97	0.97	0.96	0.95	0.94	0.94	0.93	0.92	466
CPI	0.99	0.99	0.98	0.97	0.97	0.96	0.95	0.95	0.94	0.93	0.93	0.92	466
PPI	0.99	0.99	0.98	0.98	0.97	0.96	0.96	0.95	0.95	0.94	0.93	0.93	466
WAGE	0.99	0.99	0.98	0.97	0.97	0.96	0.95	0.95	0.94	0.93	0.93	0.92	466
POP	0.99	0.99	0.98	0.98	0.97	0.97	0.96	0.96	0.95	0.95	0.94	0.93	466
LAB	0.99	0.99	0.98	0.98	0.97	0.97	0.96	0.95	0.95	0.94	0.94	0.93	454
EMP	0.99	0.99	0.98	0.98	0.97	0.96	0.96	0.95	0.95	0.94	0.93	0.93	454
UN	0.99	0.97	0.95	0.93	0.90	0.87	0.83	0.80	0.76	0.73	0.70	0.67	454
IP	0.99	0.99	0.98	0.97	0.97	0.96	0.95	0.94	0.94	0.93	0.92	0.92	466
SP500	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	466
P/E	0.98	0.96	0.95	0.92	0.90	0.88	0.85	0.83	0.81	0.79	0.77	0.75	382
D/P	0.98	0.97	0.95	0.94	0.92	0.90	0.88	0.87	0.85	0.84	0.82	0.81	466
SIG	0.63	0.52	0.41	0.39	0.39	0.34	0.30	0.27	0.29	0.27	0.26	0.25	466
GD	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	154
GNP	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	154
GNP82	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.83	0.81	0.79	0.77	0.75	154
					B. Deviations of logarithms from a time trend								
MBASE	0.99	0.98	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.92	0.91	0.90	466
BAA	0.99	0.97	0.95	0.93	0.90	0.88	0.85	0.83	0.81	0.78	0.75	0.71	466
CPI	1.00	0.99	0.99	0.98	0.98	0.97	0.97	0.96	0.95	0.95	0.94	0.93	466
PPI	1.00	0.99	0.99	0.99	0.98	0.98	0.97	0.96	0.96	0.95	0.94	0.93	466
WAGE	1.00	1.00	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.96	0.96	0.95	466
POP	0.99	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.93	0.92	0.90	0.89	466
LAB	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.91	0.90	0.89	454
EMP	0.98	0.97	0.96	0.94	0.92	0.89	0.87	0.85	0.83	0.81	0.78	0.76	454
UN	0.98	0.96	0.93	0.89	0.84	0.79	0.74	0.68	0.63	0.57	0.52	0.47	454
IP	0.99	0.96	0.93	0.89	0.85	0.81	0.77	0.73	0.69	0.65	0.61	0.57	466
SP500	0.99	0.97	0.96	0.94	0.92	0.91	0.89	0.87	0.85	0.84	0.82	0.81	466
P/E	0.97	0.94	0.90	0.87	0.83	0.79	0.75	0.72	0.68	0.65	0.62	0.59	382
D/P	0.98	0.97	0.95	0.94	0.92	0.90	0.88	0.86	0.85	0.83	0.82	0.80	466
SIG	0.61	0.50	0.38	0.36	0.36	0.31	0.27	0.23	0.26	0.23	0.23	0.21	466
GD	0.99	0.97	0.95	0.93	0.90	0.88	0.86	0.83	0.80	0.78	0.75	0.72	154
GNP	0.98	0.96	0.93	0.90	0.87	0.85	0.83	0.81	0.79	0.78	0.76	0.73	154
GNP82	0.95	0.89	0.80	0.72	0.64	0.57	0.50	0.43	0.36	0.30	0.23	0.17	154

[a]See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly. The data in panel B are residuals from a regression of the variable against an intercept and a time trend.

Table 3

Sample autocorrelations of the first differences of the logarithms of monthly and quarterly macroeconomic time series, March 1947 to December 1985 (except where noted).[a]

Series	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	Sample size T
					A. First differences of logarithms of monthly and quarterly data								
MBASE	0.17	0.25	0.32	0.26	0.27	0.33	0.18	0.29	0.33	0.28	0.29	0.32	466
BAA	0.52	0.12	−0.01	0.06	0.14	0.08	0.02	0.05	0.13	0.18	0.14	0.06	466
CPI	0.62	0.54	0.49	0.45	0.47	0.45	0.43	0.44	0.47	0.46	0.37	0.25	466
PPI	0.36	0.34	0.31	0.22	0.28	0.30	0.19	0.20	0.18	0.12	0.19	0.20	466
WAGE	0.08	0.12	0.15	0.08	0.14	0.16	0.12	0.09	0.10	0.15	0.12	0.06	466
POP	0.44	0.43	0.40	0.39	0.36	0.34	0.32	0.29	0.28	0.25	0.25	0.28	466
LAB	−0.29	0.03	−0.04	0.01	−0.03	0.00	0.12	−0.06	0.02	−0.03	0.09	−0.11	454
EMP	−0.15	0.13	0.05	0.11	0.08	−0.02	0.16	0.27	0.20	0.16	0.18	0.07	454
UN	0.13	0.32	0.21	0.18	0.19	0.07	0.03	0.03	0.00	−0.12	−0.02	−0.22	454
IP	0.44	0.27	0.18	0.12	0.02	0.01	0.04	0.02	−0.01	−0.02	−0.01	−0.21	466
SP500	0.02	−0.04	0.03	0.08	0.12	−0.07	−0.05	−0.04	0.04	−0.03	−0.02	0.06	466
P/E	−0.08	0.06	0.08	0.04	0.09	0.01	−0.07	−0.02	−0.02	−0.02	0.01	−0.11	382
D/P	0.03	−0.05	0.07	0.01	0.07	0.00	−0.07	−0.07	0.06	−0.02	−0.03	0.09	466
SIG	−0.36	0.02	−0.13	−0.02	0.06	−0.01	−0.01	−0.07	0.06	−0.03	0.01	−0.05	466
GD	0.65	0.59	0.50	0.41	0.37	0.33	0.34	0.38	0.40	0.41	0.35	0.36	154
GNP	0.45	0.28	0.02	−0.11	−0.24	−0.11	−0.03	−0.01	0.14	0.21	0.18	0.05	154
GNP82	0.37	0.25	0.00	−0.11	−0.12	−0.06	−0.03	0.08	−0.06	0.03	−0.01	−0.08	154
					B. Deviations of first differences of logarithms from a time trend								
MBASE	−0.09	0.01	0.10	0.02	0.04	0.12	−0.09	0.07	0.12	0.05	0.07	0.11	466
BAA	0.52	0.12	−0.01	0.06	0.14	0.08	0.02	0.05	0.13	0.18	0.14	0.06	466
CPI	0.55	0.45	0.40	0.35	0.37	0.34	0.32	0.32	0.36	0.34	0.23	0.09	466
PPI	0.34	0.31	0.29	0.19	0.25	0.27	0.14	0.16	0.15	0.08	0.15	0.16	466
WAGE	0.07	0.11	0.14	0.07	0.13	0.15	0.10	0.08	0.09	0.13	0.10	0.04	466
POP	0.39	0.38	0.35	0.34	0.30	0.28	0.26	0.23	0.22	0.18	0.18	0.21	466
LAB	−0.30	0.02	−0.05	0.00	−0.04	−0.01	0.12	−0.07	0.01	−0.04	0.08	−0.12	454
EMP	−0.16	0.13	0.05	0.10	0.07	−0.03	0.13	−0.10	0.13	−0.10	0.07	−0.15	454
UN	0.13	0.32	0.21	0.18	0.19	0.07	0.03	0.03	0.00	−0.12	−0.02	−0.22	454
IP	0.44	0.27	0.18	0.12	0.02	0.01	0.04	0.02	−0.01	−0.03	−0.02	−0.21	466
SP500	0.02	−0.04	0.03	0.08	0.12	−0.07	−0.05	−0.04	0.03	−0.03	−0.03	0.06	466
P/E	−0.08	0.05	0.08	0.04	0.09	0.00	−0.07	−0.02	−0.02	−0.02	0.01	−0.11	466
D/P	0.03	−0.05	0.07	0.01	0.07	0.00	−0.07	−0.07	0.06	−0.02	−0.03	0.09	466
SIG	−0.36	0.02	−0.13	−0.02	0.06	−0.01	−0.01	−0.07	0.06	−0.03	0.01	−0.05	466
GD	0.54	0.43	0.30	0.17	0.12	0.07	0.09	0.15	0.18	0.20	0.11	0.12	154
GNP	0.42	0.25	−0.03	−0.16	−0.30	−0.17	−0.07	−0.04	0.11	0.19	0.16	0.01	154
GNP82	0.36	0.24	−0.01	−0.12	−0.13	−0.07	−0.04	−0.10	−0.08	0.00	−0.03	−0.10	154

[a]See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly. The data in panel B are residuals from a regression of the variable against an intercept and a time trend.

Table 4

Sample autocorrelations of the second differences of the logarithms of monthly and quarterly macroeconomic time series, March 1947 to December 1985 (except where noted).[a]

Series	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	Sample size T
MBASE	−0.54	0.00	0.08	−0.05	−0.03	0.13	−0.16	0.05	0.06	−0.04	−0.01	0.09	466
BAA	−0.09	−0.27	−0.20	−0.01	0.14	−0.01	−0.08	−0.05	0.05	0.08	0.04	−0.04	466
CPI	−0.39	−0.05	0.00	−0.09	0.06	−0.02	−0.02	−0.05	0.06	0.11	0.04	−0.18	466
PPI	−0.48	0.00	0.06	−0.12	0.03	0.10	−0.09	0.02	0.03	−0.10	0.06	0.13	466
WAGE	−0.52	0.00	0.06	−0.07	0.02	0.04	−0.01	−0.02	−0.02	0.04	0.02	−0.06	466
POP	−0.49	0.01	−0.01	0.02	−0.01	−0.01	0.02	−0.02	0.02	−0.03	−0.03	0.05	466
LAB	−0.62	0.16	−0.05	0.03	−0.02	−0.04	0.12	−0.11	0.05	−0.06	0.12	−0.17	454
EMP	−0.63	0.16	−0.06	0.04	0.03	−0.11	0.17	−0.20	0.20	−0.17	0.17	−0.18	454
UN	−0.60	0.17	−0.05	−0.02	0.08	−0.05	−0.03	0.02	0.05	−0.13	0.18	−0.19	454
IP	−0.35	−0.08	−0.02	0.03	−0.07	−0.04	0.05	0.01	−0.01	−0.03	0.18	−0.19	466
SP500	−0.47	−0.06	0.01	0.01	0.11	−0.11	0.00	−0.03	0.07	−0.03	−0.04	0.11	466
P/E	−0.56	0.05	0.03	−0.05	0.06	0.00	−0.06	0.02	0.00	−0.01	0.07	−0.13	382
D/P	−0.46	−0.10	0.10	−0.06	0.07	0.00	−0.04	−0.07	0.11	−0.04	−0.07	0.16	466
SIG	−0.64	0.19	−0.09	0.01	0.05	−0.02	0.02	−0.07	0.08	−0.04	0.03	−0.07	466
GD	−0.42	0.04	0.01	−0.09	0.02	−0.08	−0.04	0.03	0.01	0.11	−0.11	0.03	154
GNP	−0.35	0.08	−0.12	0.01	−0.24	0.04	0.07	−0.12	0.07	0.09	0.09	−0.08	154
GNP82	−0.40	0.10	−0.11	−0.08	−0.05	0.02	0.07	−0.06	−0.05	0.10	0.02	−0.05	154

[a] See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly.

tions of the conclusion that variables like stock market volatility, inflation, or money growth are non-stationary are discussed in section 5.

Table 4 contains the first twelve autocorrelations of the second differences of the variables in table 2, which are the changes in the growth rates. For all of the series, except the bond yield *BAA*, the first autocorrelation is between −0.35 and −0.64, and most of the other autocorrelations are close to 0. This is typical of a first-order moving average process,

$$X_t = \varepsilon_t - \theta\varepsilon_{t-1}, \tag{11}$$

with $\theta \approx 0.9$. Box and Jenkins (1976) and Plosser and Schwert (1977) discuss how differencing a random series creates a first-order moving average process with $\theta = 1$, so that the first-order autocorrelation coefficient for the differences is $\rho_1 = -0.50$.

Table 5 contains average autocorrelation coefficients from 10,000 replications of a simulation experiment where the data are generated by an ARIMA(0, 1, 1) process,

$$(Y_t - Y_{t-1}) = \varepsilon_t - \theta\varepsilon_{t-1}, \qquad t = -19, \dots, T, \tag{12}$$

with $\theta = 0.8$, 0.5, 0, −0.5, or −0.8, and with samples of $T = 150$ or 450. The first twenty observations are discarded to eliminate the effects of the initial

Table 5

Average sample autocorrelations of an ARIMA(0,1,1) process, the detrended data, and the first differences of the data.[a]

Sample size T	Moving average parameter θ	Average autocorrelation coefficient at lag k											
		r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}
		A. Levels of the data											
	0.8	0.45	0.43	0.40	0.38	0.35	0.33	0.30	0.28	0.25	0.23	0.20	0.18
	0.5	0.83	0.78	0.73	0.68	0.63	0.58	0.53	0.48	0.43	0.38	0.33	0.28
150	0.0	0.94	0.88	0.82	0.76	0.70	0.64	0.59	0.53	0.47	0.41	0.35	0.30
	−0.5	0.95	0.89	0.83	0.77	0.71	0.65	0.59	0.53	0.47	0.41	0.35	0.29
	−0.8	0.96	0.90	0.84	0.78	0.72	0.66	0.60	0.54	0.48	0.42	0.36	0.31
	0.8	0.70	0.69	0.68	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.57
	0.5	0.94	0.92	0.91	0.89	0.87	0.86	0.84	0.82	0.81	0.79	0.78	0.76
450	0.0	0.98	0.96	0.95	0.93	0.91	0.89	0.88	0.86	0.84	0.83	0.81	0.79
	−0.5	0.99	0.97	0.95	0.94	0.92	0.90	0.88	0.87	0.85	0.83	0.81	0.80
	−0.8	0.99	0.97	0.95	0.94	0.92	0.90	0.88	0.87	0.85	0.83	0.81	0.80
		B. Detrended levels data											
	0.8	0.28	0.27	0.25	0.24	0.22	0.20	0.19	0.17	0.16	0.15	0.13	0.12
	0.5	0.74	0.69	0.65	0.60	0.56	0.52	0.47	0.43	0.39	0.36	0.32	0.28
150	0.0	0.93	0.87	0.81	0.75	0.69	0.64	0.58	0.53	0.48	0.43	0.39	0.34
	−0.5	0.96	0.90	0.83	0.77	0.71	0.65	0.60	0.55	0.49	0.44	0.40	0.35
	−0.8	0.97	0.90	0.84	0.77	0.71	0.66	0.60	0.55	0.50	0.45	0.40	0.35
	0.8	0.54	0.53	0.52	0.51	0.50	0.49	0.48	0.47	0.46	0.45	0.44	0.43
	0.5	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.73	0.71	0.69
450	0.0	0.98	0.95	0.93	0.91	0.89	0.87	0.85	0.83	0.81	0.79	0.77	0.75
	−0.5	0.99	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.81	0.79	0.77	0.75
	−0.8	0.99	0.97	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76
		C. First differences of the data											
	0.8	−0.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.5	−0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
150	0.0	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
	−0.5	0.39	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
	−0.8	0.48	−0.02	−0.01	−0.01	−0.01	−0.01	−0.02	−0.01	−0.01	−0.01	−0.02	−0.02
	0.8	−0.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.5	−0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
450	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	−0.5	0.40	−0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00
	−0.8	0.48	−0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	0.00

[a] These values are the averages of the sampling distribution of the autocorrelations based on 10,000 replications of an ARIMA(0,1,1) process,

$$(Y_t - Y_{t-1}) = \varepsilon_t - \theta\varepsilon_{t-1}, \qquad t = -19, \ldots, T,$$

where the first twenty observations are discarded to eliminate startup effects.

conditions $Y_{-20} = \varepsilon_{-20} = 0$. Panel A contains the average autocorrelations for the levels of the data, Y_t, panel B contains the average autocorrelations for the detrended levels data (residuals from the regression of Y_t against an intercept and a time trend), and panel C contains the average autocorrelations for the first differences $(Y_t - Y_{t-1})$. As discussed in Wichern (1973), the autocorrelations of the levels of an ARIMA(0,1,1) process are flat, and they are closer to 0 as θ is closer to 1. Detrending the data causes the autocorrelations to be smaller, even though there is no trend in the process used to generate the data. The autocorrelations for the first differences are essentially zero, except for lag 1, where $r_1 \approx -\theta/(1-\theta^2)$.

It is apparent from the simulation results in table 5 that the patterns of autocorrelations in tables 2, 3, and 4 are consistent with ARIMA(0,1,q) processes for several of the series. Stock market volatility *SIG*, the inflation rates of the *CPI*, the *PPI*, and the *GNP* deflator, the growth rate of population *POP*, and possibly the growth rate of the monetary base *MBASE* all have these characteristics.

4.2. *ARIMA(1,0,1) models for stock market volatility and growth rates of other macroeconomic variables*

Table 6 contains estimates of the ARIMA(1,0,1) model for the log of stock market volatility *SIG* (from table 2) and for the growth rates of the other variables from table 3 for the 1949–1985 sample period. The model is estimated with and without a time trend, and several tests are reported to indicate whether the autoregressive coefficient ϕ is close to unity. For example, 't-tests' of whether the AR and MA coefficients equal 1.0 are in parentheses under the coefficient estimates. Table 6 also contains estimates of the correlation between the estimates of the AR and MA parameters, $C(\phi, \theta)$, a 't-test' of whether there is parameter redundancy, $\phi = \theta$, and the Box–Pierce statistic for six lags of the residual autocorrelations $Q(6)$ to indicate the adequacy of the ARIMA model. Finally, table 6 contains estimates of an ARIMA(0,1,1) model [where the constant term in this model corresponds to the time trend coefficient β in the ARIMA(1,0,1) model], and the Box–Pierce statistic for this model, to indicate the effects of constraining the autoregressive coefficient $\phi = 1$.

There are several patterns in table 6. First, the AR and MA parameter estimates are often close to each other, usually with the AR parameter being larger. Accordingly, the correlations between the estimates $C(\phi, \theta)$ are quite large, and many of the 't-tests' for parameter redundancy are small.[5] When the

[5] The distribution of this statistic is unlikely to be Student-t, because an ARIMA(p,0,q) model is observationally equivalent to an ARIMA($p+k$,0,$q+k$) model. See Nelson and Schwert (1982) for a discussion of the effects of parameter redundancy on tests of alternative ARIMA model specifications.

Table 6

Estimates of ARIMA(1,0,1) and ARIMA(0,1,1) models for logarithms of monthly and quarterly macroeconomic time series, including tests for parameter redundancy, January 1949 to December 1985 (except where noted); t-tests under coefficient estimates.[a]

$$Y_t = \alpha + \beta[t-(T+1)/2] + \phi Y_{t-1} + u_t - \theta u_{t-1}.$$

	ARIMA(1,0,1) model						ARIMA(0,1,1) model		
Series	α	ϕ	θ	$C(\phi,\theta)$	β	$Q(6)$	θ	β	$Q(6)$
MBASE	0.00023	0.952	0.851	0.549		16.9			
	(3.46)	(−3.33)	(−4.97)	(2.54)					
	0.00383	0.078	0.171	0.985	0.014	14.6	0.871	0.028	15.9
	(3.39)	(−3.39)	(−3.05)	(−0.17)	(3.21)		(−5.50)	(1.12)	
BAA	0.01203	0.308	−0.309	0.827		10.6			
	(1.19)	(−8.54)	(−16.2)	(3.99)					
	0.01202	0.309	−0.309	0.827	−0.004	10.6	0.737	−0.519	107.0
	(1.19)	(−8.52)	(−16.2)	(3.99)	(−0.05)		(−8.15)	(−0.22)	
CPI	0.00023	0.938	0.589	0.582		20.5			
	(2.82)	(−3.23)	(−8.68)	(5.77)					
	0.00037	0.896	0.532	0.706	0.001	16.4	0.661	0.017	27.6
	(3.26)	(−3.52)	(−8.21)	(4.52)	(2.05)		(−9.49)	(0.43)	
PPI	0.00025	0.925	0.725	0.773		15.3			
	(2.12)	(−2.73)	(−5.32)	(2.67)					
	0.00029	0.911	0.711	0.817	0.001	15.6	0.788	0.018	13.7
	(2.17)	(−2.64)	(−4.97)	(2.28)	(1.02)		(−7.22)	(0.29)	
WAGE	0.00737	−0.673	−0.694	0.993		46.1			
	(5.36)	(−5.49)	(−5.67)	(0.04)					
	0.00762	−0.728	−0.740	0.996	0.011	30.8	0.929	0.004	8.5
	(4.82)	(−4.89)	(−4.98)	(0.02)	(2.87)		(−4.00)	(0.29)	
POP	0.00006	0.949	0.740	0.698		0.3			
	(2.44)	(−2.49)	(−5.81)	(3.42)					
	0.00008	0.938	0.727	0.752	0.000	0.3	0.796	0.001	1.1
	(2.41)	(−2.44)	(−5.49)	(2.98)	(0.87)		(−7.08)	(0.15)	
LAB	0.00133	0.089	0.357	0.960		2.8			
	(4.85)	(−5.31)	(−3.99)	(−0.81)					
	0.00121	0.152	0.438	0.951	0.002	3.3	0.976	0.005	33.3
	(5.04)	(−5.48)	(−3.99)	(−0.98)	(2.59)		(−2.18)	(1.04)	
EMP	0.00212	−0.471	−0.333	0.985		19.8			
	(4.82)	(−6.01)	(−5.10)	(−0.27)					
	0.00209	−0.480	−0.340	0.984	0.003	18.4	0.949	0.006	26.4
	(4.86)	(−6.14)	(−5.18)	(−0.28)	(1.45)		(−3.36)	(0.56)	
UN	−0.00004	0.820	0.594	0.895		12.0			
	(−0.01)	(−2.90)	(−4.67)	(1.56)					
	−0.00019	0.819	0.593	0.895	0.012	12.0	0.729	−0.237	7.7
	(−0.05)	(−2.90)	(−4.66)	(1.55)	(0.37)		(−8.21)	(−0.08)	
IP	0.00120	0.638	0.251	0.885		3.4			
	(2.65)	(−4.61)	(−7.56)	(2.25)					
	0.00121	0.637	0.251	0.886	−0.002	3.5	0.606	0.033	23.6
	(2.65)	(−4.61)	(−7.54)	(2.23)	(−0.67)		(−10.4)	(0.16)	
SP500	0.00769	−0.290	−0.335	0.998		11.6			
	(1.49)	(−1.72)	(−1.81)	(0.03)					
	0.00792	−0.328	−0.373	0.998	−0.017	11.4	0.979	−0.030	13.3
	(1.53)	(−1.77)	(−1.86)	(0.03)	(−0.74)		(−1.96)	(−0.66)	

Table 6 (continued)

	ARIMA(1,0,1) model						ARIMA(0,1,1) model		
Series	α	ϕ	θ	$C(\phi,\theta)$	β	$Q(6)$	θ	β	$Q(6)$
P/E	0.00165	−0.236	−0.165	0.997		8.5			
	(0.04)	(−1.76)	(−1.63)	(−0.05)					
	0.01302	−0.235	−0.164	0.997	−0.135	8.4	0.998	−0.025	9.3
	(0.21)	(−1.76)	(−1.63)	(−0.05)	(−0.29)		(−0.18)	(−0.09)	
DP	−0.00941	−0.548	−0.631	0.991		3.5			
	(−0.65)	(−5.11)	(−5.80)	(0.14)					
	0.00954	−0.565	−0.646	0.991	0.085	3.3	0.989	0.118	7.8
	(−0.65)	(−5.26)	(−5.99)	(0.14)	(0.74)		(−1.13)	(0.79)	
SIG	−0.55683	0.842	0.368	0.726		10.0			
	(−4.21)	(−4.22)	(−9.81)	(4.99)					
	−0.65472	0.815	0.342	0.755	0.161	9.2	0.559	−0.054	23.2
	(−4.39)	(−4.40)	(−9.63)	(4.55)	(1.94)		(−11.2)	(−0.01)	
GD	0.00047	0.955	0.627	0.579		0.8			
	(1.24)	(−1.44)	(−4.57)	(3.18)					
	0.00111	0.897	0.574	0.743	0.011	1.4	0.665	−0.020	0.7
	(1.66)	(−1.76)	(−4.04)	(2.10)	(1.37)		(−5.27)	(−0.14)	
GNP	0.00850	0.534	0.168	0.903		4.8			
	(2.69)	(−2.82)	(−4.25)	(1.04)					
	0.00934	0.489	0.173	0.917	0.045	7.8	0.697	−0.126	23.1
	(2.70)	(−2.80)	(−3.90)	(0.82)	(1.93)		(−4.95)	(−0.41)	
GNP82	0.00370	0.498	0.167	0.915		5.9			
	(2.40)	(−2.78)	(−3.98)	(0.87)					
	0.00379	0.487	0.158	0.920	−0.008	5.9	0.623	−0.126	16.1
	(2.36)	(−2.73)	(−3.90)	(0.83)	(−0.43)		(−5.66)	(−0.36)	

[a] See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are measured monthly. The log of stock market volatility *SIG* from table 2, and the growth rates of the other variables from table 3 are used to estimate the models. $Q(6)$ is the Box–Pierce (1970) test for residual autocorrelation based on six lags, which should be distributed as χ^2 with four degrees of freedom for the ARIMA(1,0,1) model and with five degrees of freedom for the ARIMA(0,1,1) model. $C(\phi,\theta)$ is the estimate of the correlation between the estimates of the AR parameter ϕ and the MA parameter equal 1.0 (which will not have a Student-t distribution under the null hypothesis). Under the estimates of α and β are t-tests of whether these coefficients equal zero. Under the correlation $C(\phi,\theta)$ is a 't-test' of whether there is parameter redundancy ($\phi=\theta$). The estimates of the trend coefficient β are multiplied by 1000.

AR coefficient is constrained to equal unity in the ARIMA(0,1,1) model, the estimates of the MA coefficient θ are never lower than 0.56 and in many cases are very close to 1. The t-tests on the time trend coefficients are usually small, although there are a few cases (*MBASE*, *CPI*, *WAGE*, and *LAB*) where the t-tests for a deterministic trend are large for the ARIMA(1,0,1) model, but not for the ARIMA(0,1,1) model. The Box–Pierce statistics $Q(6)$ indicate that there is substantial autocorrelation in the residuals from these models for several series. It is interesting to note, however, that this diagnostic test is often smaller for the ARIMA(0,1,1) than for the ARIMA(1,0,1) model, even though the latter model allows an additional free parameter. It seems that constraining the AR parameter $\phi = 1$ is sometimes helpful when $\phi \approx \theta$.

Thus, the estimates of ARIMA(1,0,1) and ARIMA(0,1,1) models in table 6 illustrate the argument that these models often describe the behavior of the growth rates of macroeconomic time series. The 'typical' ARIMA(0,1,1) model for growth rates (and stock market volatility) has a large positive moving average coefficient θ. The simulations and test results below show that for this case, the unit root tests of stationarity proposed by Dickey, Fuller, Phillips, Perron, and Said are misleading.

4.3. Regression t-tests for unit roots

While the autocorrelations in table 2 suggest that most of the series contain at least one unit root in the autoregressive polynomial, and the autocorelations in table 3 suggest that several of the series of growth rates still contain an autoregressive unit root, the autocorrelations do not provide a formal test. Table 7 contains the 0.05 level critical values for six different 't-tests' where the underlying data are assumed to be generated by an ARIMA(0,1,1) process, as in table 5. These values are interpolated from the simulation results in Schwert (1987). Panel A contains the critical values for tests with no time trend in the model, and panel B contains the critical values for the tests where a time trend is included. The AR(1) test is based on eqs. (1) or (7) with $p = 1$; the Phillips corrections to the AR(1) test use eqs. (6) or (10) with l lags of the residual autocorrelations in (5), where l is set in (13a) and (13b); the ARMA(1,1) tests use eqs. (3) or (8); the AR(l_4) and AR(l_{12}) tests use eqs. (2) or (7) with l_4 and l_{12} lags, respectively, where l_4 and l_{12} are defined in

$$l_4 = \text{Int}\{4(T/100)^{1/4}\}, \tag{13a}$$

$$l_{12} = \text{Int}\{12(T/100)^{1/4}\}, \tag{13b}$$

so that $l_4 = 5$, $l_{12} = 17$ when $T = 444$, and $l_4 = 4$, $l_{12} = 13$ when $T = 140$.[6]

As discussed by Schwert (1987), it is apparent from table 7 that the critical values for these tests do not conform to the Student-t distribution, and they are sensitive to the process which generates the data. In particular, when the underlying process is ARIMA(0,1,1) with $\theta = 0.8$, the Dickey–Fuller (1979) test and the tests proposed by Phillips (1987) and Phillips and Perron (1986) have critical values that are far below the Dickey–Fuller distributions tabulated in Fuller (1976) [which are represented by the AR(1) test when the data are generated with $\theta = 0$]. In other words, these tests will lead to the conclusion that economic data are stationary much too frequently. The best test, in the sense that it is least affected by the different processes used to generate the data, is the high-order autoregressive test proposed by Said and Dickey (1984),

[6]The number of lags used in the high-order autoregressions and in Phillips' (1987) tests are allowed to grow with the sample size, as suggested by Said and Dickey (1984).

Table 7

0.05 critical values for regression τ-tests for a unit root in the autoregressive polynomial of the ARIMA model, where the simulated data follow an ARIMA(0,1,1) model.[a]

A. No time trend in model, τ_μ

Sample size T	Moving average parameter θ	AR(1)	$Z_{\tau\mu}(l_4)$	$Z_{\tau\mu}(l_{12})$	ARMA(1,1)	AR(l_4)	AR(l_{12})
	0.8	−9.98	−10.16	−11.15	−2.98	−4.38	−2.92
	0.5	−5.45	−5.30	−6.16	−2.78	−3.02	−2.82
140	0.0	−2.90	−2.93	−2.95	−2.83	−2.87	−2.82
	−0.5	−2.59	−2.73	−2.67	−2.87	−2.93	−2.85
	−0.8	−2.59	−2.74	−2.67	−3.03	−3.02	−2.87
	0.8	−13.29	−13.81	−17.78	−3.98	−4.54	−2.92
	0.5	−5.72	−4.81	−5.97	−3.44	−2.94	−2.83
444	0.0	−2.89	−2.91	−2.95	−3.05	−2.88	−2.84
	−0.5	−2.55	−2.77	−2.78	−2.98	−2.84	−2.84
	−0.8	−2.53	−2.75	−2.75	−3.00	−2.73	−2.83

B. Time trend in model, τ_τ

Sample size T	Moving average parameter θ	AR(1)	$Z_{\tau\tau}(l_4)$	$Z_{\tau\tau}(l_{12})$	ARMA(1,1)	AR(l_4)	AR(l_{12})
	0.8	−10.94	−11.06	−11.69	−3.19	−5.09	−3.49
	0.5	−6.58	−6.59	−7.36	−2.66	−3.61	−3.36
140	0.0	−3.47	−3.53	−3.47	−3.12	−3.41	−3.36
	−0.5	−2.80	−3.15	−2.96	−3.30	−3.49	−3.36
	−0.8	−2.77	−3.13	−2.95	−3.43	−3.61	−3.38
	0.8	−15.07	−15.68	−19.27	−3.21	−5.43	−3.47
	0.5	−7.06	−6.36	−8.04	−3.25	−3.54	−3.39
444	0.0	−3.44	−3.48	−3.54	−3.43	−3.42	−3.40
	−0.5	−2.79	−3.25	−3.20	−3.46	−3.37	−3.38
	−0.8	−2.71	−3.22	−3.18	−3.49	−3.22	−3.39

[a] These values are interpolated from tables 2 and 3 in Schwert (1987). They are the 0.05 fractiles of the sampling distribution of the regression 't-test' for a unit root against the alternative hypothesis that the process is stationary around a constant mean (panel A), or that the process is stationary around a time trend (panel B). Based on 10,000 replications of an ARIMA(0,1,1) process,

$$(Y_t - Y_{t-1}) = \varepsilon_t - \theta\varepsilon_{t-1}, \qquad t = -19,\ldots,T,$$

where the first twenty observations are discarded to eliminate startup effects. The AR(1) test is based on eqs. (1) or (7) with $p = 1$; the Phillips corrections to the AR(1) test $Z_\tau(l)$ use eqs. (6) or (10) with l lags of the residual autocovariances in (5), where l is defined in (13a) and (13b); the ARMA(1,1) tests use eqs. (3) or (8); the AR(l_4) and AR(l_{12}) tests use eqs. (2) or (7) with l_4 and l_{12} lags, respectively, where l_4 and l_{12} are defined in (13a) and (13b).

AR(l_{12}). All of these tests are applied to the series in table 1 to determine whether inferences about stationarity are different from different tests. Based on the results of these simulations, and the evidence in tables 2 and 3, the Dickey–Fuller and Phillips tests of stationarity will reject a unit root for the series that have small positive autocorrelations which do not decay.

Table 8 contains τ-tests for unit roots in the levels of the variables (similar to table 2), both with (panel A) and without (panel B) a time trend in the model. Thus, the null hypothesis is that these variables follow a non-stationary process, and the alternative hypothesis is that the variables follow a stationary process (around a deterministic time trend in panel B). All of the tests are computed as in table 7, except that the ARMA(1, $l_4 - 1$) test estimates the same number of parameters as the AR(l_4) test. In table 8, tests that reject the unit root hypothesis for all of the values of the moving average parameter in table 7 are indicated with a plus ($^+$), and tests that reject using the Dickey–Fuller critical value (i.e., with $\theta = 0$), but not for other values of the moving average parameter are indicated with an asterisk (*) (these are cases where the outcome of the test depends on the process that generated the data).

In panel A of table 8, all of the tests fail to reject the hypothesis that the series are non-stationary, except for the tests on stock market volatility. For volatility, the AR(1) and Phillips tests would reject the unit root hypothesis strongly using the Dickey–Fuller critical values, but not using the critical values for the case where $\theta = 0.8$ in table 7. The ARMA(1, $l_4 - 1$) test does not reject a unit root in the autoregressive polynomial, while the AR(l_4) and AR(l_{12}) tests both reject the unit root hypothesis for all values of θ in table 7.

In panel B of table 8, a few more of the unit root tests are ambiguous. The tests on stock market volatility have the same conclusions as in panel A, except the AR(l_4) test is now ambiguous (i.e., it would reject a unit root except when $\theta = 0.8$ in table 7). In addition, the AR(1) and Phillips' $Z_{\tau\tau}(l_4)$ test would reject the unit root hypothesis for the population series *POP* using the Dickey–Fuller critical values, but not using the critical values for the case where $\theta = 0.8$ in table 7. The ARMA(1, $l_4 - 1$) test rejects a unit root in the autoregressive polynomial, while the AR(l_4) and AR(l_{12}) tests do not reject the unit root hypothesis for all values of θ in table 7. Also, the AR(l_4) test rejects the unit root hypothesis for the unemployment rate for some but not all of the moving average parameters θ in table 7.

Thus, even for series like the unemployment rate (*UN*), the nominal bond yield (*BAA*), the dividend yield (*D/P*) and the price/earnings ratio (*P/E*), the tests in table 8 do not reject the unit root hypothesis. For stock market volatility (*SIG*), the conclusion of the test depends on the type of ARIMA model that generated the data. The autocorrelations of *SIG* in table 3 look like the autocorrelations of an ARIMA(0, 1, 1) process with $\theta \approx 0.8$ in table 5, which suggests that the appropriate critical values to use in table 8 are for the case where $\theta = 0.8$. In this case, the unit root hypothesis should not be rejected

Table 8

Regression τ-tests for the presence of a unit root in the autoregressive polynomial of the ARIMA model for the logarithms of macroeconomic time series, January 1949 to December 1985.[a,b]

Series	AR(1)	$Z_{\tau\mu}(l_4)$	$Z_{\tau\mu}(l_{12})$	ARMA$(1, l_4 - 1)$	AR(l_4)	AR(l_{12})
A. No time trend in model, τ_μ, Dickey–Fuller 0.05 critical value = −2.90						
MBASE	10.23	9.79	7.34	9.45	6.04	2.12
BAA	−0.51	−0.76	−0.92	−0.77	−0.94	−1.29
CPI	10.51	5.46	3.56	6.09	2.54	1.22
PPI	4.10	2.41	1.58	2.59	1.39	0.80
WAGE	3.64	3.09	2.25	3.06	2.45	1.67
POP	5.09	2.95	1.98	3.12	1.41	0.40
LAB	1.53	2.25	2.27	2.45	2.39	1.57
EMP	0.98	0.92	0.77	0.81	0.75	1.00
UN	−1.28	−1.97	−2.24	−1.96	−2.83	−2.05
IP	−1.37	−1.15	−1.15	−1.00	−1.18	−1.21
SP500	−1.69	−1.67	−1.68	−1.58	−1.63	−1.68
P/E	−1.80	−1.91	−1.95	−1.90	−2.07	−1.55
D/P	−2.15	−2.21	−2.19	−2.13	−2.27	−2.02
SIG	−9.83*	−9.88*	−11.98*	−1.88	−4.82+	−3.13+
GD	7.03	4.06	2.81	4.06	1.58	0.39
GNP	3.16	2.50	2.55	1.86	2.08	1.91
GNP82	−1.14	−0.99	−1.05	−1.05	−0.87	−1.30

Series	AR(1)	$Z_{\tau\tau}(l_4)$	$Z_{\tau\tau}(l_{12})$	ARMA$(1, l_4 - 1)$	AR(l_4)	AR(l_{12})
B. Time trend in model, τ_τ, Dickey–Fuller 0.05 critical value = −3.44						
MBASE	−1.15	−1.16	−1.05	−1.16	−1.12	−0.85
BAA	−1.46	−2.10	−2.54	−2.00	−2.37	−3.16
CPI	−0.47	−0.54	−0.69	−0.64	−0.60	−0.92
PPI	−1.27	−1.18	−1.25	−1.29	−1.19	−1.21
WAGE	−0.59	−0.66	−0.82	−0.68	−0.73	−1.03
POP	−5.59*	−3.81*	−3.15	−3.87+	−2.85	−2.65
LAB	−2.49	−2.57	−2.59	−2.57	−2.65	−2.61
EMP	−1.83	−1.90	−2.08	−1.93	−2.15	−2.07
UN	−2.46	−3.13	−3.44	−2.97	−4.06*	−3.25
IP	−1.69	−2.57	−2.59	−2.64	−3.31	−2.29
SP500	−2.38	−2.46	−2.44	−2.37	−2.54	−2.25
P/E	−2.67	−2.82	−2.87	−2.62	−2.99	−2.54
D/P	−2.08	−2.14	−2.10	−2.07	−2.20	−1.92
SIG	−10.27*	−10.38*	−12.28*	−1.90	−5.17*	−3.49+
GD	−1.43	−1.18	−1.18	−0.91	−1.85	−2.03
GNP	−1.27	−1.30	−1.29	−1.12	−1.48	−1.37
GNP82	−1.57	−2.08	−1.95	−2.36	−2.31	−1.70

[a] See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly.

[b] Superscript plus signs indicate tests where one would reject the unit root hypothesis for all of the processes in table 7 at the 0.05 level. Asterisks indicate tests where one would reject the unit root hypothesis using the Dickey–Fuller critical value, but not using the critical values for all of the processes in table 7 at the 0.05 level.

for the stock market volatility, even though conventional Dickey–Fuller tests would reject non-stationarity.

Table 9 contains τ-tests for unit roots in the first differences of the variables (similar to table 3), both with (panel A) and without (panel B) a time trend in the model. Thus, the null hypothesis is that the growth rates of these variables follow a non-stationary process, and the alternative hypothesis is that the growth rates follow a stationary process (around a deterministic time trend in panel B). As in table 8, tests that reject the unit root hypothesis for all of the values of the moving average parameter in table 7 are indicated with a plus ($^+$), and tests that reject using the Dickey–Fuller critical value (i.e., with $\theta = 0$), but not for other values of the moving average parameter are indicated with an asterisk (*) (these are cases where the outcome of the test depends on the process that generated the data).

In panel A of table 9, most of the tests reject the unit root hypothesis using the Dickey–Fuller critical values ($\theta = 0$ in table 7), although the ARMA(1, $l_4 - 1$) test does not reject a unit root in the autoregressive polynomial except for the growth rate of stock market volatility *SIG* and the growth rate of the monetary base *MBASE*. For the *CPI* and the *GNP* deflator inflation rates, the only tests that reject the unit root hypothesis depend on the type of process that generated the data. Like the level of stock market volatility in table 7, these inflation rates have autocorrelations that are similar to an ARIMA(0, 1, 1) process with $\theta \approx 0.8$, in which case the unit root hypothesis would not be rejected using the critical values in table 7. The highest-order autoregressive tests AR(l_{12}) does not reject the unit root hypothesis for money growth (*MBASE*), population growth (*POP*), and GNP growth (*GNP*).

The tests in panel B of table 9 that include a time trend yield similar conclusions to the results in panel A. The ARMA(1, $l_4 - 1$) test rejects a unit root in the autoregressive polynomial only for the growth rate of stock market volatility. Several of the other series yield results that are ambiguous, depending on the structure of the process that generated the data. Thus, for *CPI* and *GNP* deflator inflation rates and for population growth, the conclusion of the test depends on the type of ARIMA model that generated the data. The autocorrelations of these series in table 4 look like the autocorrelations of an ARIMA(0, 1, 1) process with $\theta \approx 0.8$ in table 5, which suggests that the appropriate critical values to use in table 7 are for the case where $\theta = 0.8$. In this case, the unit root hypothesis should not be rejected, even though conventional Dickey–Fuller tests would reject the unit root hypothesis. For several of the other series, the conclusion of the test procedure depends on which test is used.

4.4. Tests for unit roots based on the normalized bias, $T(\hat{\rho} - 1)$

As discussed above, Dickey and Fuller (1979) note that the normalized bias $T(\hat{\rho} - 1)$ provides a more powerful test against the hypothesis of stationarity

Table 9

Regression τ-tests for the presence of a unit root in the autoregressive polynomial of the ARIMA model for the first differences of the logarithms of macroeconomic time series, January 1949 to December 1985.[a,b]

Series	AR(1)	$Z_{\tau\mu}(l_4)$	$Z_{\tau\mu}(l_{12})$	ARMA(1, $l_4 - 1$)	AR(l_4)	AR(l_{12})
A. No time trend in model, τ_μ, Dickey–Fuller 0.05 critical value = −2.90						
MBASE	−18.46⁺	−19.38⁺	−24.04⁺	−3.47*	−5.31⁺	−2.29
BAA	−11.69*	−11.32*	−11.91*	−2.26	−7.11⁺	−4.34⁺
CPI	−9.25*	−9.14*	−11.75*	−2.40	−4.05*	−2.91*
PPI	−14.23⁺	−15.06⁺	−18.18⁺	−2.09	−5.31⁺	−3.95⁺
WAGE	−19.86⁺	−20.27⁺	−22.78⁺	−1.88	−6.70⁺	−3.63⁺
POP	−13.11*	−14.00⁺	−17.69*	−2.06	−4.73⁺	−2.65
LAB	−26.78⁺	−27.45⁺	−26.71⁺	−2.58	−10.55⁺	−3.91⁺
EMP	−23.66⁺	−23.48⁺	−23.82⁺	−2.61	−6.94⁺	−5.71⁺
UN	−16.07⁺	−17.09⁺	−17.89⁺	−2.69	−6.01⁺	−5.27⁺
IP	−13.12*	−13.37*	−13.09*	−2.46	−7.97⁺	−6.04⁺
SP500	−20.33⁺	−20.37⁺	−20.34⁺	−2.17	−7.63⁺	−5.53⁺
P/E	−20.24⁺	−20.22⁺	−20.21⁺	−1.44	−6.86⁺	−5.72⁺
D/P	−20.07⁺	−20.07⁺	−20.05⁺	−0.97	−8.37⁺	−5.72⁺
SIG	−30.00⁺	−35.91⁺	−48.07⁺	−4.27⁺	−13.45⁺	−6.97⁺
GD	−5.24*	−5.07*	−6.46*	−1.44	−2.40	−1.38
GNP	−7.76*	−7.72*	−8.00*	−1.16	−5.60⁺	−2.77
GNP82	−8.21*	−8.14*	−7.83*	−1.80	−6.04⁺	−4.10⁺
B. Time trend in model, τ_τ, Dickey–Fuller 0.05 critical value = −3.44						
Series	AR(1)	$Z_{\tau\tau}(l_4)$	$Z_{\tau\tau}(l_{12})$	ARMA(1, $l_4 - 1$)	AR(l_4)	AR(l_{12})
MBASE	−23.32⁺	−23.21⁺	−23.64⁺	−2.82	−8.35⁺	−3.45*
BAA	−11.67*	−11.29*	−11.89*	−2.25	−7.09*	−4.30⁺
CPI	−10.71*	−10.98*	−13.71*	−2.14	−4.62*	−3.42*
PPI	−14.81*	−15.66*	−18.45*	−1.87	−5.60⁺	−4.25⁺
WAGE	−20.46⁺	−20.72⁺	−22.59⁺	−1.60	−7.14⁺	−4.05⁺
POP	−13.74*	−14.70*	−18.13*	−1.99	−4.96*	−2.68
LAB	−27.06⁺	−28.13⁺	−28.03⁺	−2.74	−11.07⁺	−4.53⁺
EMP	−23.74⁺	−23.55⁺	−23.80⁺	−2.91	−7.01⁺	−5.90⁺
UN	−16.07⁺	−17.09⁺	−17.88*	−2.69	−6.00⁺	−5.26⁺
IP	−13.13*	−13.37*	−13.03*	−2.45	−7.99⁺	−6.10⁺
SP500	−20.34⁺	−20.37⁺	−20.34⁺	−2.12	−7.65⁺	−5.59⁺
P/E	−20.22⁺	−20.20⁺	−20.19⁺	−1.43	−6.85⁺	−5.73⁺
D/P	−20.07⁺	−20.06⁺	−20.04⁺	−0.93	−8.38⁺	−5.76⁺
SIG	−29.96⁺	−35.87⁺	−48.01⁺	−4.25⁺	−13.44⁺	−6.97⁺
GD	−6.59*	−6.68*	−7.93*	−1.78	−3.24	−1.80
GNP	−8.21*	−8.05*	−7.83*	−1.86	−6.14⁺	−3.59⁺
GNP82	−8.20*	−8.13*	−7.78*	−1.64	−6.04⁺	−4.23⁺

[a] See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly.

[b] Superscript plus signs indicate tests where one would reject the unit root hypothesis for all of the processes in table 7 at the 0.05 level. Asterisks indicate tests where one would reject the unit root hypothesis using the Dickey–Fuller critical value, but not using the critical values for all of the processes in table 7 at the 0.05 level.

($\rho < 1$) than the regression 't-test'. Table 10 contains the 0.05 level critical values for eight different tests where the underlying data are assumed to be generated by an ARIMA(0,1,1) process, as in table 5. These values are interpolated from the simulation results in Schwert (1987). Panel A contains the critical values for tests with no time trend in the model, and panel B contains the critical values for the tests where a time trend is included. The AR(1) test is based on eqs. (1) or (7) with $p = 1$; the Phillips corrections to the AR(1) test use eqs. (4) or (9) with l lags of the residual autocorrelations in (5), where l is set in (13a) and (13b); the ARMA(1,1) tests use eqs. (3) or (8); the AR(l_4) and AR(l_{12}) tests use eqs. (2) or (7) with l_4 and l_{12} lags, respectively, where l_4 and l_{12} are defined in (13a) and (13b). The AR(l_4)^c and AR(l_{12})^c tests use the correction suggested by Fuller (1976), $Tc(\hat{\rho} - 1)$, where $c = 1/(1 - \phi'_1 - \cdots - \phi'_{l-1})$, and the coefficients ϕ'_i are the coefficients of the lagged differences DY_{t-i} in (2). It is apparent from table 10 that the critical values for these tests depend on the process that generates the data. The values for $\theta = 0$ are essentially the same as those tabulated by Fuller (1976); otherwise, the critical values for this test are inversely related to the moving average parameter θ. With $\theta = 0.8$, the critical values are well below the Dickey–Fuller critical values. In other words, these tests would lead to the conclusion that economic data are stationary too frequently when the data follow an ARIMA(0,1,1) process with $\theta \approx 0.8$. Even with the Fuller correction $Tc(\hat{\rho} - 1)$ none of the autoregressive tests or the Phillips tests are well-behaved. The ARMA(1,1) test, which is based on the ARIMA model that generated the data in the experiments, comes closest to having the same critical values for all values of the moving average parameter θ for both sample sizes.

Table 11 contains normalized bias tests for unit roots in the levels of the variables (similar to the τ-tests in table 8), both with (panel A) and without (panel B) a time trend in the model. All of the tests are computed as in table 10, except that the ARMA($1, l_4 - 1$) test estimates the same number of parameters as the AR(l_4) test. In table 11, tests that reject the unit root hypothesis for all of the values of the moving average parameter in table 10 are indicated with a plus ($^+$), and tests that reject using the Dickey–Fuller critical value (i.e., with $\theta = 0$), but not for other values of the moving average parameter are indicated with an asterisk (*) (these are cases where the outcome of the test depends on the process that generated the data). In panel A of table 11, all of the tests fail to reject the hypothesis that the series are non-stationary, except for the tests on stock market volatility *SIG* and the adjusted AR(l_4)^c test for the unemployment rate *UN*. For volatility, the AR(1) and Phillips' tests would reject the unit root hypothesis strongly using the Dickey–Fuller critical values, but not using the critical values for the case where $\theta = 0.8$ in table 10. The high-order autoregressive tests reject for all values of θ in table 10.

In panel B of table 11, a few more of the unit root tests are ambiguous. The tests on stock market volatility have the same conclusions as in panel A,

Table 10

0.05 critical values for tests for a unit root in the autoregressive polynomial of the ARIMA model based on the normalized bias of the root estimate, where the simulated data follow an ARIMA(0,1,1) model.[a]

A. No time trend in model, $T(\hat{\rho}_\mu - 1)$

Sample size T	Moving average parameter θ	AR(1)	$Z_{\rho\mu}(l_4)$	$Z_{\rho\mu}(l_{12})$	ARMA(1,1)	AR(l_4)	AR(l_4)^c	AR(l_{12})	AR(l_{12})^c
	0.8	−116.9	−130.1	−196.4	−47.1	−83.3	−62.3	−72.9	−50.1
	0.5	−48.5	−44.9	−65.9	−17.4	−29.2	−19.9	−31.1	−36.4
140	0.0	−13.9	−14.3	−14.2	−14.7	−14.4	−16.6	−16.1	−35.8
	−0.5	−8.2	−11.9	−11.1	−13.7	−9.8	−17.4	−10.8	−37.6
	−0.8	−7.8	−11.9	−10.9	−13.7	−8.8	−19.4	−9.2	−37.3
	0.8	−252.8	−285.4	−556.4	−45.8	−108.6	−48.5	−75.1	−21.6
	0.5	−60.6	−41.0	−66.8	−19.5	−29.1	−16.3	−29.9	−19.2
444	0.0	−14.2	−14.4	−14.7	−15.7	−14.3	−15.2	−14.9	−18.6
	−0.5	−8.4	−12.9	−12.8	−14.9	−9.5	−14.7	−10.1	−19.2
	−0.8	−7.6	−12.5	−12.6	−14.4	−7.6	−13.1	−8.3	−18.8

B. Time trend in model, $T(\hat{\rho}_\tau - 1)$

Sample size T	Moving average parameter θ	AR(1)	$Z_{\rho\tau}(l_4)$	$Z_{\rho\tau}(l_{12})$	ARMA(1,1)	AR(l_4)	AR(l_4)^c	AR(l_{12})	AR(l_{12})^c
	0.8	−130.4	−143.7	−204.3	−79.7	−108.8	−106.2	−113.9	−148.3
	0.5	−65.4	−65.8	−90.5	−28.3	−44.2	−33.8	−52.8	−108.3
140	0.0	−20.9	−21.8	−21.0	−22.6	−22.4	−28.3	−27.8	−114.9
	−0.5	−12.2	−17.6	−14.9	−20.9	−15.6	−30.4	−19.0	−110.8
	−0.8	−11.3	−17.4	−14.4	−20.5	−13.9	−34.4	−16.1	−115.9
	0.8	−301.6	−350.7	−637.1	−48.1	−153.5	−76.2	−115.0	−41.6
	0.5	−89.5	−70.4	−119.8	−24.7	−45.4	−26.5	−48.4	−37.9
444	0.0	−21.7	−22.2	−23.1	−22.5	−22.3	−24.1	−24.2	−37.2
	−0.5	−12.5	−19.2	−18.7	−21.9	−14.6	−23.0	−16.0	−36.4
	−0.8	−11.4	−18.8	−18.5	−21.4	−11.8	−20.4	−13.5	−36.8

[a] These values are interpolated from tables 4 and 5 in Schwert (1987). They are the 0.05 fractiles of the sampling distribution of the normalized bias $T(\hat{\rho} - 1)$ against the alternative hypothesis that the process is stationary around a constant mean (panel A), or that the process is stationary around a time trend (panel B). Based on 10,000 replications of an ARIMA(0,1,1) process,

$$(Y_t - Y_{t-1}) = \varepsilon_t - \theta\varepsilon_{t-1}, \qquad t = -19,\dots,T,$$

where the first twenty observations are discarded to eliminate startup effects. The AR(1) test is based on eqs. (1) or (7) with $p = 1$; the Phillips corrections to the AR(1) test $Z_\rho(l)$ use eqs. (4) or (9) with l lags of the residual autocovariances in (5), where l is defined in (13a) and (13b); the ARMA(1,1) tests use eqs. (3) or (8); the AR(l_4) and AR(l_{12}) tests use eqs. (2) or (7) with l_4 and l_{12} lags, respectively, where l_4 and l_{12} are defined in (13a) and (13b).

except all of the tests are now ambiguous (i.e., they would reject a unit root except when $\theta = 0.8$ in table 10). In addition, three of the tests are ambiguous for the unemployment rate *UN*, and one test is ambiguous for both industrial production *IP* and the *P/E* ratio for the S&P portfolio.

Thus, the results in table 11 are similar to the τ-test results in table 8. Even for series like the unemployment rate (*UN*) and the nominal bond yield

Table 11

Tests for the presence of a unit root in the autoregressive polynomial of the ARIMA model for the logarithms of macroeconomic time series based on the normalized bias of the root estimate, January 1949 to December 1985.[a,b]

A. No time trend in model, $T(\hat{\rho}_\mu - 1)$, *Dickey–Fuller* 0.05 *critical value* = −14.2

Series	AR(1)	$Z_{\rho\mu}(l_4)$	$Z_{\rho\mu}(l_{12})$	ARMA(1, l_4 − 1)	AR(l_4)	AR(l_4)[c]	AR(l_{12})	AR(l_{12})[c]
MBASE	1.48	1.48	1.47	1.48	1.23	1.47	0.53	1.31
BAA	−0.55	−1.17	−1.72	−1.22	−0.84	−1.71	−1.18	−3.40
CPI	1.53	1.50	1.46	1.52	0.34	1.33	0.17	1.08
PPI	1.32	1.24	1.12	1.29	0.42	1.06	0.25	0.81
WAGE	0.58	0.58	0.57	0.58	0.41	0.56	0.29	0.54
POP	0.41	0.40	0.39	0.41	0.10	0.34	0.03	0.18
LAB	0.59	0.63	0.63	0.63	0.92	0.65	0.66	0.61
EMP	0.45	0.44	0.42	0.42	0.34	0.43	0.46	0.51
UN	−3.39	−7.88	−10.14	−8.34	−7.00	−17.43*	−5.49	−10.73
IP	−0.81	−1.04	−1.04	−1.01	−0.63	−1.31	−0.62	−0.86
SP500	−2.23	−2.33	−2.26	−2.37	−2.16	−2.44	−2.20	−2.04
P/E	−6.42	−7.26	−7.52	−8.22	−7.49	−9.01	−5.92	−5.59
D/P	−7.41	−7.99	−7.77	−8.48	−7.86	−8.81	−7.24	−7.46
SIG	−159.37*	−161.44*	−255.62*	−23.50*	−96.05*	−55.44+	−76.12+	−30.28+
GD	1.12	1.10	1.07	1.01	0.26	0.98	0.09	0.59
GNP	0.55	0.55	0.55	0.47	0.36	0.55	0.38	0.55
GNP82	−0.46	−0.51	−0.49	−0.65	−0.33	−0.50	−0.52	−0.57

B. Time trend in model, $T(\hat{\rho}_\tau - 1)$, *Dickey–Fuller* 0.05 *critical value* = −21.8

Series	AR(1)	$Z_{\rho\tau}(l_4)$	$Z_{\rho\tau}(l_{12})$	ARMA(1, l_4 − 1)	AR(l_4)	AR(l_4)[c]	AR(l_{12})	AR(l_{12})[c]
MBASE	−0.65	−0.64	−0.73	−0.66	−0.63	−0.66	−0.47	−0.84
BAA	−4.19	−8.74	−12.79	−9.10	−5.76	−12.18	−8.29	−35.08
CPI	−0.21	−0.44	−0.83	−0.48	−0.20	−0.75	−0.31	−1.78
PPI	−0.99	−1.45	−2.12	−1.56	−0.84	−2.02	−0.84	−2.61
WAGE	−0.56	−0.72	−1.20	−0.75	−0.68	−0.93	−0.96	−1.86
POP	−4.20	−4.67	−5.38	−4.62	−1.99	−5.53	−1.86	−7.66
LAB	−6.58	−4.90	−4.82	−4.36	−6.77	−4.63	−6.57	−5.40
EMP	−5.68	−6.21	−7.66	−7.11	−6.56	−8.44	−6.34	−7.89
UN	−8.71	−16.73	−20.88*	−17.15	−13.41*	−34.36*	−12.56	−31.15
IP	−5.62	−13.09	−13.36	−16.29	−10.13	−23.43*	−7.78	−13.26
SP500	−8.17	−8.98	−8.76	−9.81	−8.73	−10.19	−7.91	−8.04
P/E	−12.68	−14.38	−14.92	−15.80	−14.46*	−17.93	−13.21	−13.71
D/P	−7.23	−7.77	−7.47	−8.30	−7.72	−8.63	−7.00	−7.13
SIG	−171.34*	−175.96*	−264.76*	−30.96*	−108.50*	−64.60*	−91.53*	−40.51*
GD	−0.86	−1.17	−1.60	−0.84	−0.91	−3.07	−1.09	−6.60
GNP	−1.66	−2.08	−1.93	−2.12	−1.83	−2.88	−1.77	−2.89
GNP82	−5.12	−8.89	−7.81	−14.67	−7.28	−12.33	−6.50	−10.07

[a]See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly.

[b]Superscript plus signs indicate tests where one would reject the unit root hypothesis for all of the processes in table 10 at the 0.05 level. Asterisks indicate tests where one would reject the unit root hypothesis using the Dickey–Fuller critical value, but not using the critical values for all of the processes in table 10 at the 0.05 level.

Table 12

Tests for the presence of a unit root in the autoregressive polynomial of the ARIMA model for the first differences of the logarithms of macroeconomic time series based on the normalized bias of the root estimate, January 1949 to December 1985.[a,b]

Series	AR(1)	$Z_{\rho\mu}(l_4)$	$Z_{\rho\mu}(l_{12})$	ARMA(1, $l_4 - 1$)	AR(l_4)	AR(l_4)[c]	AR(l_{12})	AR(l_{12})[c]
A. No time trend in model, $T(\hat{\rho}_\mu - 1)$, Dickey–Fuller 0.05 critical value = −14.2								
MBASE	−381.14+	−497.71+	−952.32+	−25.33*	−166.90+	−64.80+	−75.12+	−8.45
BAA	−212.18*	−193.30*	−223.59+	−566.37+	−204.29+	−164.54+	−191.06+	−190.42+
CPI	−140.82*	−136.56*	−248.53*	−16.73*	−67.05*	−32.87*	−53.55*	−21.81
PPI	−278.12+	−336.44+	−588.01+	−30.39*	−139.83+	−68.97+	−128.19+	−67.04+
WAGE	−418.24+	−497.72+	−775.12+	−18.61*	−260.67+	−131.22+	−182.01+	−49.85*
POP	−248.01*	−302.36+	−547.17*	−20.10*	−112.79+	−52.03*	−75.20+	−18.72
LAB	−539.79+	−507.51+	−544.08*	−15.10*	−623.78+	−1035.58+	−407.96+	−71.29+
EMP	−488.18+	−571.48+	−665.23+	−259.43+	−311.02+	−147.56+	−390.80+	−468.07+
UN	−323.90+	−413.59+	−478.30*	−136.12+	−178.90+	−96.37+	−257.57+	−2481.90+
IP	−248.43*	−263.89*	−246.55*	−352.42+	−226.22+	−223.79+	−308.06+	−274.99+
SP500	−429.41+	−445.08+	−434.98*	−321.99+	−346.66+	−198.57+	−459.86+	−683.88+
P/E	−384.77+	−425.78+	−419.06*	−210.00+	−286.42+	−162.08+	−425.43+	−193.08+
D/P	−423.73+	−422.34+	−404.04*	−320.32+	−373.86+	−253.35+	−472.23+	−603.16+
SIG	−592.20+	−491.00+	−439.42*	−513.91+	−1007.24+	−1558.63+	−1745.39+	−143.15+
GD	−46.53*	−42.82*	−75.65*	−7.59	−23.00*	−12.98	−14.29*	−5.04
GNP	−83.76*	−82.38*	−92.81*	−53.76+	−84.42+	−94.01+	−67.29*	−43.38*
GNP82	−89.99*	−87.00*	−70.44*	−37.01+	−94.34+	−117.90+	−119.36+	−121.84+
B. Time trend in model, $T(\hat{\rho}_\tau - 1)$, Dickey–Fuller 0.05 critical value = −21.8								
Series	AR(1)	$Z_{\rho\tau}(l_4)$	$Z_{\rho\tau}(l_{12})$	ARMA(1, $l_4 - 1$)	AR(l_4)	AR(l_4)[c]	AR(l_{12})	AR(l_{12})[c]
MBASE	−485.94+	−515.55+	−667.16+	−243.55+	−400.01+	−263.36+	−234.11+	−39.61*
BAA	−212.16*	−192.87*	−223.35*	−566.55+	−204.21+	−164.42+	−190.63+	−187.87+
CPI	−181.79*	−194.63*	−333.43*	−26.05*	−94.38*	−49.41*	−83.18*	−42.29+
PPI	−294.66*	−358.25+	−563.76*	−36.98*	−157.25+	−80.86+	−154.87+	−108.44+
WAGE	−432.17+	−498.92+	−727.69+	−23.37*	−294.49+	−160.60+	−225.18+	−77.04+
POP	−266.40*	−328.73*	−565.91*	−27.35*	−127.59*	−60.88*	−86.85*	−23.06
LAB	−544.61+	−500.01+	−503.09*	−586.90+	−674.48+	−1807.12+	−543.67+	−171.93+
EMP	−490.11+	−570.64+	−651.00+	−321.82+	−316.76+	−152.09+	−416.83+	−305.12+
UN	−324.38+	−413.82+	−478.21*	−137.15+	−179.35+	−96.73+	−258.27+	−2682.15+
IP	−249.08*	−263.66*	−242.87*	−349.72+	−227.43+	−226.04+	−312.89+	−259.80+
SP500	−430.19+	−444.00+	−428.38*	−324.32+	−349.42+	−201.54+	−473.20+	−515.01+
P/E	−384.89+	−425.27+	−417.64*	−210.36+	−286.82+	−162.52+	−429.41+	−187.20+
D/P	−424.22+	−421.28+	−398.82*	−322.25+	−375.15+	−254.99+	−478.08+	−538.05+
SIG	−592.23+	−490.34+	−438.64*	−518.63+	−1007.45+	−1556.84+	−1747.16+	−142.91+
GD	−67.57*	−70.18*	−108.28*	−19.53	−40.41*	−25.63*	−31.86*	−16.76
GNP	−90.33*	−83.10*	−71.79*	−42.48*	−95.72*	−118.83+	−104.74*	−1567.86+
GNP82	−90.62*	−87.35*	−69.71*	−37.75*	−96.61*	−125.49+	−136.34+	−77.46+

[a]See table 1 for a definition of the variables. The ending point for all series is December 1985. *GD*, *GNP*, and *GNP82* are measured quarterly; other series are monthly.

[b]Superscript plus signs indicate tests where one would reject the unit root hypothesis for all of the processes in table 10 at the 0.05 level. Asterisks indicate tests where one would reject the unit root hypothesis using the Dickey–Fuller critical value, but not using the critical values for all of the processes in table 10 at the 0.05 level.

(*BAA*), the tests do not reject the unit root hypothesis. For stock market volatility (*SIG*), the conclusion of the test depends on the type of ARIMA model that generated the data.

Table 12 contains normalized bias tests for unit roots in the first differences of the variables (similar to the τ-tests in table 9). As in table 11, tests that reject the unit root hypothesis for all of the values of the moving average parameter in table 10 are indicated with a plus ($^+$), and tests that reject using the Dickey–Fuller critical value (i.e., with $\theta = 0$), but not for other values of the moving average parameter are indicated with an asterisk (*) (cases where the outcome of the test depends on the process that generated the data).

In panel A of table 12, most of the tests reject the unit root hypothesis using the Dickey–Fuller critical values ($\theta = 0$ in table 10). For the *CPI* and the *GNP* deflator inflation rates, none of the tests would reject the unit root hypothesis for an ARIMA(0,1,1) process with $\theta \approx 0.8$. In addition, the highest-order adjusted autoregressive test AR$(l_{12})^c$ does not reject the unit root hypothesis for the growth in the monetary base (*MBASE*) or for population growth (*POP*).

The tests in panel B of table 12 that include a time trend yield similar conclusions to the results in panel A. For *CPI* and *GNP* deflator inflation rates and for population growth, the conclusion of the test depends on the type of ARIMA model that generated the data. The autocorrelations of these series in table 4 look like the autocorrelations of an ARIMA(0,1,1) process with $\theta \approx 0.8$ in table 5, which suggests that the appropriate critical values to use in table 10 are for the case where $\theta = 0.8$. In this case, the unit root hypothesis should not be rejected, even though conventional Dickey–Fuller tests would reject the unit root hypothesis. For several of the tests for the other series, the conclusion of the test procedure depends on which test is used.

4.5. Summary of test results

Several patterns emerge from the test statistics in tables 8, 9, 11, and 12. First, for the levels (of the logarithms) of the data, almost all of the series behave like integrated processes. Virtually all of the tests in tables 8 and 11 are consistent with a unit root in the autoregressive part of the ARIMA model. The only exception to this rule is for stock market volatility *SIG*, where the results of the unit root test depend on the type of ARIMA model that generated the data.

Second, for the growth rates (the first differences) of the data, there are several series where the unit root tests are ambiguous, depending on the ARIMA model that generated the data. For these series, where the autocorrelations of the growth rates in table 3 are small and positive for many lags (like the inflation rate of the *CPI* or of the *GNP* deflator, and for population growth), the tests in tables 9 and 12 suggest that there is another unit root in the autoregressive polynomial. Using the critical values implied for an ARIMA(0, 1, 1) process with $\theta = 0.8$, most of the tests in table 9 and 12 do not reject the unit root hypothesis for these growth rates. In these cases, conventional Dickey–Fuller tests and most of the variants developed by Phillips (1987), Phillips and Perron (1986), and Said and Dickey (1984) would lead to false conclusions.

Finally, when a mixed ARMA model is used to conduct the unit root tests, the results are more consistent with a unit root in the autoregressive part of the model for the growth rates (first differences) than the tests based on a pure autoregressive representation of the data. Given the numerous reasons to believe that economic time series contain moving average components, this possible evidence of unit roots in the growth rates of so many variables is noteworthy.

5. Consequences of unit root tests for economic modeling

As discussed by Nelson and Plosser (1982) and many subsequent authors, the conclusion that an economic time series contains a unit root in the autoregressive polynomial of its ARIMA representation has important consequences for dynamic economic models. For example, with a unit root there is no deterministic long-run growth path to which the economic variable tends to revert. Moreover, uncertainty about the level of an economic series grows larger indefinitely as one forecasts further into the future. Thus, for an integrated series (containing a unit root), it is not meaningful to discuss the 'long-run' mean or variance of the process. In terms of business cycle modeling, a unit root means that part of the innovation to the series causes a permanent change in the level of the series.

Following Nelson and Plosser, many authors have found that aggregate output series (such as *GNP*, *IP*, *GNP82*, *POP*, *LAB* and *EMP*), aggregate price level series (such as *CPI*, *PPI*, and *GD*), and other aggregate nominal series (such as *MBASE*, *WAGE*, and *SP500*) contain a unit root. What is perhaps surprising about the results in tables 8 and 11 is that variables that are expressed as percentages or ratios, such as the bond yield *BAA*, the unemployment rate *UN*, the price/earnings ratio *P/E* and dividend yield *D/P* for the S&P composite portfolio, and the volatility of stock returns *SIG*, also seem to be non-stationary.[7]

[7]Series like these have been studied in detail by Shiller (1981), Poterba and Summers (1986), and others, often with the conclusion that the series are stationary using conventional Dickey–Fuller tests.

Even more surprising is the evidence that growth rate series, such as inflation in the *CPI* or the *GNP* deflator and population growth, may contain a unit root.[8] If these growth rates follow ARIMA(0, 1, 1) processes, then the permanent effect of this period's innovation on the level of future growth rates Y_{t+l} is

$$E(Y_{t+l} \mid Y_t, \dots) = E(Y_{t+l} \mid Y_{t-1}, \dots) + (1-\theta)u_t \quad \text{for} \quad l > 0. \tag{14}$$

Based on the estimates of the ARIMA(0, 1, 1) model in table 6, these series all have moving average coefficients θ close to 1, so that the permanent effect is a small fraction of the total innovation. Nevertheless, to the extent that one concludes that series like interest rates, inflation rates, or stock market volatility follow integrated ARIMA processes, it is not meaningful to talk about a 'long-run' or 'steady-state' level for these series.

The source of the nonstationarity in these variables is worthy of further consideration. For example, technological change that is non-stationary could cause series such as the *CPI* to seem non-stationary merely because the Bureau of Labor Statistics fails to adjust for changes in 'quality' accurately.[9] On the other hand, such non-stationary 'measurement errors' in the price level should not induce non-stationary behavior in inflation rates or nominal interest rates. Perhaps the *CPI* inflation rate is non-stationary because the money growth rate is non-stationary; such a conjecture is the basis for tests of 'co-integration' [i.e., a common unit root in two series, such that a regression of one non-stationary series on another yields stationary errors, see Engle and Granger (1987) and Stock and Watson (1987)]. In fact, it is possible that all of these factors play a role in causing the observed behavior of the CPI, although it would be difficult to identify the relative importance of different sources of non-stationarity without additional theory and data.

To the extent that changing demographic characteristics or measurement practices cause non-stationarity in observed economic time series, most economists would properly ignore such behavior, since it has little to do with their economic theories. On the other hand, if non-stationarity results from integrated processes for technology or tastes, economists who are interested in

[8]Although Fama (1975) and Nelson and Schwert (1977) model the inflation rate and the short-term nominal interest rate as series containing a unit root. There is even some evidence that is weakly consistent with the hypothesis that the real interest rate (the nominal rate minus inflation) contains a unit root.

[9]To the extent that innovations are not perfectly correlated with the depreciation of the existing stock, technology is an integrated process.

(long-run) growth models or (short-term) business cycle models can make serious errors is using data to calibrate their misspecified theoretical constructs. Only careful analysis of the data, including knowledge of the measurement practices used to construct the data, can hope to resolve these questions.

6. Summary

This paper examines the time series behavior of seventeen important macroeconomic variables. Sample autocorrelations of the logarithms, and the first and second differences of the logs of these variables suggest that many of these variables are generated by non-stationary ARIMA processes. There are numerous economic and statistical reasons to believe that economic time series contain moving average components. Schwert (1987) shows that, when conducting formal tests of non-stationarity (unit root tests), it is important to consider whether the underlying process contains a moving average component, since the distribution of the unit root test statistics can be far different from the distributions reported by Fuller (1976). Even the asymptotically correct extensions suggested by Said and Dickey (1984, 1985), Phillips (1987), and Phillips and Perron (1986) are affected by the process generating the data in large finite samples. In particular, if the series is generated by an ARIMA(0, 1, 1) process with a large moving average parameter (where the sample autocorrelations of the data are small and positive for many lags), most of the tests considered depart substantially from the distributions calculated by Dickey and Fuller. Estimates of ARIMA(0, 1, 1) models for the growth rates of the variables studied in this paper indicate that this model may describe many monthly or quarterly macroeconomic variables. In that case, use of the Dickey–Fuller critical values would reject the hypothesis of non-stationarity far too often.

The unit root tests developed by Dickey and Fuller, and the extensions mentioned above, are applied to the levels and first differences of the seventeen economic time series. In cases where it seems that the series is generated by mixed ARIMA processes with large moving average coefficients, such as the monthly *CPI* inflation rate or the log of the monthly standard deviation of returns to the S&P composite portfolio, the unit root tests reject non-stationarity using the Dickey–Fuller critical values, but not using the critical values calculated by Schwert (1987). Thus, the conclusions of these unit root tests critically depend on the assumption that the underlying process is a pure autoregressive model. Given the important implications that non-stationarity can have for economic modeling, one should consider the correct specification of the ARIMA process before testing for the presence of a unit root in the autoregressive polynomial.

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