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Heteroskedasticity in Stock Returns

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ABSTRACT

We use predictions of aggregate stock return variances from daily data to estimate time-varying monthly variances for size-ranked portfolios. We propose and estimate a single factor model of heteroskedasticity for portfolio returns. This model implies time-varying betas. Implications of heteroskedasticity and time-varying betas for tests of the capital asset pricing model (CAPM) are then documented. Accounting for heteroskedasticity increases the evidence that risk-adjusted returns are related to firm size. We also estimate a constant correlation model. Portfolio volatilities predicted by this model are similar to those predicted by more complex multivariate generalized-autoregressive-conditional-heteroskedasticity (GARCH) procedures.

MANY RESEARCHERS HAVE NOTED that the variance of aggregate stock returns changes over time. For example, French, Schwert, and Stambaugh (1987) use daily returns to the Standard & Poor's (S&P) composite portfolio to estimate monthly volatility from 1928 to 1984. They estimate that the standard deviation of aggregate monthly returns was about four times larger in the 1929–1933 period than in the 1953–1970 period. This paper (i) investigates the relation between aggregate volatility and the variance of monthly returns to disaggregated portfolios of stocks and (ii) examines the effect of portfolio heteroskedasticity on some common empirical tests in finance.

We start with a model which implies that the conditional covariance is a quadratic function of the conditional market standard deviation,

$$\text{cov}_{t-1}(R_{it}, R_{jt}) = a_{0ij} + a_{1ij}\,\sigma_{t-1}(R_{et}) + a_{2ij}\,\sigma^2_{t-1}(R_{et}), \tag{1}$$

where cov_{t-1} is the conditional covariance operator and $\sigma^2_{t-1}(R_{et})$ is the conditional variance of aggregate stock returns in period t based on information in period $t-1$. We empirically investigate two special cases of (1). Section I analyzes a single index model of stock return heteroskedasticity,

$$\text{cov}_{t-1}(R_{it}, R_{jt}) = a_{0ij} + a_{2ij}\,\sigma^2_{t-1}(R_{et}). \tag{2}$$

If $a_{2ij} = 0$, there is no evidence of changes in portfolio variances and covariances related to the volatility of the market portfolio R_{et}. If $a_{0ij} = 0$ and $a_{2ij} \neq 0$,

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portfolio volatility is proportional to aggregate volatility. If both a_{0ij} and a_{2ij} are nonzero, portfolio returns are heteroskedastic, but the relation to aggregate volatility is nonproportional. Glejser (1969) refers to the situation where the standard deviation or variance of residuals is not proportional to the regressor as "mixed" heteroskedasticity.

Aside from its obvious parsimony, the model in (2) is consistent with at least one model of security returns. Harvey (1989) develops a conditional capital asset pricing model (CAPM) which can be written as

$$\sum_{i=1}^{N} \omega_i R_{it} R_{jt} = a_j \sigma_{t-1}^2 (R_{et}) + \eta_{jt}. \tag{3}$$

Our model is consistent with this conditional CAPM if each term in the weighted sum on the left-hand side is linear in $\sigma_{t-1}^2(R_{et})$.

Section I analyzes the single index model of heteroskedasticity in (2) for monthly returns to size-ranked portfolios from 1927 to 1986. It also analyzes the implications of (2) for the "market model" regression equation. In particular, nonproportional heteroskedasticity implies that portfolios have beta coefficients that vary over time. Section I also shows how tests of the Sharpe (1964)-Lintner (1965) CAPM are affected by the use of a weighted least squares (WLS) estimation procedure that accounts for heteroskedasticity. It also shows how the use of time-varying betas implied by the single index model in (2) affects tests of the CAPM. Relative to tests that assume constant betas over time and that ignore heteroskedasticity, these new tests find stronger evidence that small firms earn higher average returns than implied by the CAPM.

Section II estimates a constrained, constant correlation version of the heteroskedasticity model (1) where the time-varying covariance matrix $\mathbf{\Sigma}_t$ is

$$\mathbf{\Sigma}_t = \mathbf{S}_t' \boldsymbol{\rho} \mathbf{S}_t, \tag{4}$$

$\mathbf{S}_t$ is a diagonal matrix containing the time-varying standard deviations of the portfolio returns on the diagonal, and $\boldsymbol{\rho}$ is the time-invariant correlation matrix. The elements of $\mathbf{S}_t$ are represented as linear functions of the market standard deviation,

$$\sigma_{t-1}(R_{it}) = s_{0i} + s_{1i}\sigma_{t-1}(R_{et}). \tag{5}$$

The advantage of this specification is that a smaller number of time-varying parameters must be estimated. Also, there are statistical reasons to prefer estimates of time-varying standard deviations rather than variances. In terms of (1), the constant correlation model in (4) and (5) implies that $a_{0ij} = \rho_{ij} s_{0i} s_{0j}$, $a_{1ij} = \rho_{ij}(s_{0i}s_{1j} + s_{1i}s_{0j})$, and $a_{2ij} = \rho_{ij} s_{1i} s_{1j}$, where ρ_{ij} is the unconditional correlation coefficient between R_{it} and R_{jt}.

Section III compares the results of the regression models for heteroskedasticity with univariate and multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models for the size-ranked portfolio returns. Section IV presents brief conclusions and suggestions for future work.

I. A Single Index Model for Heteroskedasticity

A. Estimates of Aggregate Stock Return Volatility

To model heteroskedasticity in monthly stock returns, we use estimates of aggregate stock return volatility derived from daily returns to the Standard & Poor's composite portfolio from 1928 to 1986. We use returns to a weighted average of the Dow-Jones Industrial and Transportation portfolios for 1926–1927. (See Schwert (1990b) for a description of these data.) As argued by Merton (1980), if stock prices behave like a geometric random walk, the variance of returns can be measured better by using more frequent observations. Following French, Schwert, and Stambaugh (1987), we calculate the monthly variance of the S&P return s_t^2 using

$$s_t^2 = \sum_{i=1}^{N_t} r_i^2 + 2 \sum_{i=1}^{N_t-1} r_i r_{i+1}, \tag{6}$$

where there are N_t daily returns r_i in month t. The second summation allows for first-order autocorrelation of portfolio returns due to nonsynchronous trading. (See Fisher (1966) or Scholes and Williams (1977).)[1]

Table I contains weighted least squares estimates of a 12th-order autoregression for s_t from 1927–1986,

$$s_t = \beta_0 + \sum_{i=1}^{12} \beta_i s_{t-i} + u_t. \tag{7a}$$

It also contains estimates of a regression of the standard deviation of the excess monthly return to the CRSP equally weighted portfolio of New York Stock Exchange (NYSE) stocks s_{et} on 12 lags of s_t,

$$s_{et} = \beta_0 + \sum_{i=1}^{12} \beta_i s_{t-i} + u_t. \tag{7b}$$

We use the absolute excess monthly return $|R_{et} - R_{ft}|$, minus its sample mean $\hat{\mu}_e$, multiplied by the constant $(\pi/2)^{1/2}$ to estimate the standard deviation of the CRSP equally weighted portfolio s_{et}. If excess returns have a normal distribution with a constant mean and time-varying standard deviation, the transformed variable, $(\pi/2)^{1/2}\{|R_{et} - R_{ft}| - \hat{\mu}_e\}$, has an expected value equal to the standard deviation of the excess returns. Equation (8) is a regression of the absolute errors

[1] In (6) we ignore daily average returns in estimating the monthly variance. We have also calculated versions of s_t^2 that subtract the monthly sample mean and/or ignore the effects of first-order autocorrelation. None of these corrections has an important effect on the time series behavior of market volatility. The S&P return does not include dividends. We have calculated variances for returns to the Center for Research in Security Prices (CRSP) value-weighted portfolio of New York and American Stock Exchange stocks, including dividends, from 1962 to 1986, and there is no substantive difference between the CRSP and S&P measures of aggregate volatility. Thus, the omission of dividends is unlikely to be important in the earlier part of the sample.

Table I

Estimates of Conditional Standard Deviations of Monthly Stock Market Returns Using 12 Lags of the Standard Deviation of the Standard & Poor's Composite Portfolio Based on Daily Returns in the Month, 1927–1986

Twelve lags of the monthly standard deviation of the S&P composite portfolio based on the daily returns within the month, s_{t-i}, are used to forecast the current S&P standard deviation (s_t) in column 2 or the standard deviation of the CRSP equally weighted portfolio of NYSE stocks $s_{et} = (\pi/2)^{1/2}|R_{et} - R_{ft} - \hat{\mu}_e|$ in column 3. R_{et} is the monthly return to the CRSP portfolio, R_{ft} is the yield on a one-month Treasury security, and $\hat{\mu}_e$ is the average excess return ($R_{et} - R_{ft}$) for the sample period. These equations are estimated using iterated weighted least squares (WLS), where the absolute residuals, $|\hat{u}_t|$, are regressed against the prediction from (7a) or (7b) $\hat{\sigma}_{st}$ in the Glejser regression (8), and then (7a) or (7b) is reestimated using WLS, and then (8) is estimated using WLS, and so forth. This procedure, recommended by Davidian and Carroll (1987), is repeated three times. The sum of the 12 lag coefficients ($\beta_1 + \cdots + \beta_{12}$) has a t-test for whether the sum equals one in parentheses below it. R^2 is the coefficient of determination in terms of the unweighted data. The Box-Pierce (1970) statistic for 24 lags of the autocorrelations of the unweighted residuals, which should be distributed as χ^2 with 12 degrees of freedom, has its p-value in parentheses below it.

$$s_t = \beta_0 + \sum_{i=1}^{12} \beta_i s_{t-i} + u_t, \tag{7a}$$

$$s_{et} = \beta_0 + \sum_{i=1}^{12} \beta_i s_{t-i} + u_t, \tag{7b}$$

$$|\hat{u}_t| = \gamma_0 + \gamma_1 \hat{\sigma}_{st} + \epsilon_t. \tag{8}$$

Lag, β_i	Daily S&P, s_t	Monthly CRSP Equally Weighted, s_{et}
constant	.0051	.0038
	(3.30)	(.87)
1	.4620	.4944
	(9.68)	(3.95)
2	.1365	.1822
	(2.80)	(1.38)
3	−.0070	.0397
	(−.16)	(.32)
4	.0921	.1362
	(2.07)	(1.11)
5	−.0081	.0590
	(−.20)	(.49)
6	.0006	.2597
	(.01)	(2.09)
7	.0790	−.1161
	(1.91)	(−1.06)
8	.0680	.1141
	(1.57)	(.96)
9	−.0302	−.1374
	(−.72)	(−1.20)
10	.0795	.3953
	(1.84)	(3.09)
11	−.0072	−.0554
	(−.17)	(−.49)
12	.0263	−.1150
	(.72)	(−1.25)
Sum of 12 lags	.8914	1.2566
(t-test = 1)	(−2.57)	(2.19)
R^2	.567	.270
Box-Pierce	17.1	31.0
(p-value)	(.1441)	(.0020)

from (7a) or (7b) $|\hat{u}_t|$ against the prediction from (7a) or (7b) $\hat{\sigma}_{st}$, a formulation suggested by Glejser (1969):

$$|\hat{u}_t| = \gamma_0 + \gamma_1 \hat{\sigma}_{st} + \epsilon_t. \tag{8}$$

As recommended by Davidian and Carroll (1987), we estimate (7a) or (7b) by least squares, and then estimate (8) by WLS, and then estimate (7a) or (7b) by WLS, and so forth, for a total of three iterations. Davidian and Carroll (1987) argue that standard deviation specifications such as these are more robust than variance specifications based on s_t^2 or s_{et}^2. Thus, the last 12 monthly values of the S&P standard deviation based on daily data are used to predict the standard deviation for month t. Schwert (1989) uses many additional financial and macroeconomic variables that are measured on a monthly basis and finds little evidence that these data help to predict aggregate stock volatility beyond an autoregression similar to Table I.

The results in Table I support Black's (1976) intuition that there is a serially correlated factor causing stock market volatility to change over time. The pattern of the lag coefficients in Table I is similar for both regressions, although the coefficients are larger for the monthly equally weighted standard deviations and the coefficient of determination R^2 is lower. This is to be expected. The S&P standard deviation uses about 22 daily returns, rather than one monthly return for the CRSP equally weighted portfolio. Thus, the S&P standard deviation has less measurement error. Since the S&P portfolio is weighted more heavily toward large firms, the risk of this portfolio is likely to be lower.

Compared with monthly data, daily data provide a large advantage in estimating volatility. Since the daily S&P standard deviation measures the "true" standard deviation with much less error than the monthly equally weighted measure, it is preferable to use its lagged values to predict future volatility in other stock portfolios.

We use the fitted values from (7b), the regression with the equally weighted market portfolio, as the regressor $\hat{\sigma}_{et}$ in the remainder of the paper because the size-ranked portfolios used in the subsequent tests are equally weighted. We use the simpler notation $\hat{\sigma}_{et}$ to denote the estimate of the conditional standard deviation of aggregate stock returns $\sigma_{t-1}(R_{et})$ throughout the remainder of the paper.

Since we use the predictions from Table I as regressors in the subsequent analysis, there is a "generated regressors" problem (see Pagan (1984)). This is another reason to use the estimates of monthly volatility from daily data in constructing the predictions $\hat{\sigma}_{et}$. Better predictions make the generated regressor problem less serious. This is especially clear in calculating standard errors for regressions containing generated regressors using the technique of Murphy and Topel (1985), where the covariance matrix of the parameters from Table I increases the estimates of the standard errors of the second-step regression coefficients. In short, if $\hat{\sigma}_{et}$ can be estimated more precisely, the correction to the standard errors in the subsequent tests is reduced, and the power of these tests is correspondingly increased.

B. Estimates of the Single Index Model

Table II contains estimates of (2) for five equally weighted size-ranked portfolios of NYSE common stock returns from 1927 to 1986.[2] There are 15 variances and covariances among the five portfolio returns. To estimate the parameters in (2), we use the regression:

$$(R_{it} \cdot R_{jt}) = a_{0ij} + a_{2ij}\hat{\sigma}^2_{et} + u_{ijt},$$
$$t = 1, \cdots, T; \quad i = 1, \cdots, 5; \quad j = i, \cdots, 5, \qquad (9)$$

where R_{it} is the excess return of portfolio i in period t (the return minus the yield on a short-term government security), less the sample mean of the series. Portfolio 1 contains the smallest 20 percent of firms based on the market value of equity at the beginning of the year, and portfolio 5 contains the largest 20 percent. The dependent variable is an estimate of the covariance of security i with security j in period t. Since the conditional covariance is a function of the conditional means, $\text{cov}_{t-1}(R_{it}, R_{jt}) = E_{t-1}(R_{it} \cdot R_{jt}) - E_{t-1}(R_{it}) \cdot E_{t-1}(R_{jt})$, significant variation in conditional mean returns that is related to the regressor in (9) would cause this regression to be misspecified. Auxiliary regressions of excess returns on $\hat{\sigma}^2_{et}$ and on $\hat{\sigma}_{et}$, not reported here, show that there is no strong relation between conditional mean excess returns and aggregate conditional volatility for these portfolios.

Table II also contains estimates of the Glejser (1969) regression for the errors:

$$\hat{u}^2_{ijt} = \alpha_{0ij} + \alpha_{2ij}\hat{\sigma}^2_{et} + e_{ijt}, \quad t = 1, \cdots, T, \qquad (10)$$

where $\alpha_{2ij} = 0$ implies that there is no heteroskedasticity in (9), and $\alpha_{2ij} > 0$ implies that the errors in (9) are heteroskedastic. Where the Glejser regression (10) shows heteroskedasticity in (9), the least squares standard errors for equations (9) and (10) are biased downward. Accordingly, we present t-statistics using heteroskedasticity consistent standard errors from Hansen (1982) in parentheses below the OLS coefficient estimates. Since the regressors in (9) and (10) are functions of estimates from Table I, we also present standard errors that correct for the generated regressors problem in brackets, using the technique of Murphy and Topel (1985). Table III contains WLS estimates of (9), where each observation is weighted by dividing by the square root of the fitted value from (10). As in Table I, we iterate three times between (9) and (10) using WLS each time.

The results in Table II show a strong relation between the covariances of monthly returns and the predicted stock market variance $\hat{\sigma}^2_{et}$. The estimates of the slope coefficients $\hat{a}_{2ij}$ are large (between 1.0 and 4.0), and they are all reliably larger than zero. The estimates of the intercepts $\hat{a}_{0ij}$ are all negative, but they are small relative to their standard errors. (The largest t-statistic is -1.94.) The slope coefficients are largest for the covariances involving the small firm portfolios, and they are smaller for the larger firm portfolios, reflecting the fact that the unconditional variances and covariances of small firm portfolios are larger. (This is also reflected in the residual standard deviations $S(\hat{u})$.) Black (1976,

[2] We also performed all of the tests in this paper on (i) five value-weighted size-ranked portfolios of NYSE stocks and (ii) 12 equally and 12 value-weighted industry portfolios of NYSE stocks. Since the results are similar, we do not report them.

Table II

Least Squares Estimates of the Single Index Model for Time-Varying Covariances Among Monthly Returns to Size-Ranked Portfolios, 1927–1986

Equation (9) is estimated using products of excess returns to equally weighted size-ranked portfolios, where R_{1t} is the excess return to the smallest firm portfolio and R_{5t} is the excess return to the portfolio of the largest firms. The predicted variance of the CRSP equally weighted portfolio return $\hat{\sigma}^2_{et}$ is from Table I, R^2 is the coefficient of determination, $S(\hat{u})$ is the residual standard deviation, and $SR(\hat{u})$ is the Studentized range of the residuals. t-statistics in parentheses use Hansen's (1982) heteroskedasticity consistent standard errors, with 12 lags of the residuals and the regressors and a damping factor of 0.7. (See the RATS computer manual for details.) t-statistics in brackets correct the Hansen standard errors for the fact that the regressor $\hat{\sigma}^2_{et}$ is a function of the predictions from Table I, using the technique of Murphy and Topel (1985). The Glejser regressions (10) estimate the relation between the variance of the residuals from (9) and the predicted market variance $\hat{\sigma}^2_{et}$.

$$(R_{it} \cdot R_{jt}) = a_{0ij} + a_{2ij}\hat{\sigma}^2_{et} + u_{ijt}, \tag{9}$$

$$\hat{u}^2_{ijt} = \alpha_{0ij} + \alpha_{2ij}\hat{\sigma}^2_{et} + e_{ijt}. \tag{10}$$

Portfolios i, j	a_{0ij}	a_{2ij}	R^2	$S(\hat{u})$	$SR(\hat{u})$	α_{0ij}	α_{2ij}
1, 1	−.0071	3.9843	.182	.0613	21.52	−.0075	2.2204
	(−1.94)	(4.37)				(−2.61)	(3.04)
	[−1.83]	[3.75]				[−2.53]	[2.81]
1, 2	−.0046	2.7657	.223	.0375	20.89	−.0028	.8350
	(−1.82)	(4.44)				(−2.68)	(3.12)
	[−1.71]	[3.80]				[−2.59]	[2.88]
1, 3	−.0044	2.5210	.251	.0316	19.65	−.0020	.5977
	(−1.82)	(4.22)				(−2.71)	(3.19)
	[−1.73]	[3.65]				[−2.62]	[2.92]
1, 4	−.0034	2.0447	.247	.0259	21.11	−.0014	.4024
	(−1.74)	(4.36)				(−2.62)	(3.15)
	[−1.65]	[3.74]				[−2.54]	[2.89]
1, 5	−.0032	1.7816	.247	.0226	23.10	−.0011	.3255
	(−1.82)	(4.00)				(−2.58)	(2.99)
	[−1.73]	[3.50]				[−2.51]	[2.77]
2, 2	−.0028	2.0302	.266	.0245	18.99	−.0012	.3488
	(−1.53)	(4.54)				(−2.74)	(3.31)
	[−1.44]	[3.85]				[−2.65]	[3.02]
2, 3	−.0029	1.9155	.286	.0219	16.88	−.0010	.2872
	(−1.60)	(4.32)				(−2.75)	(3.40)
	[−1.51]	[3.71]				[−2.66]	[3.08]
2, 4	−.0021	1.5703	.278	.0184	17.82	−.0007	.2003
	(−1.45)	(4.48)				(−2.68)	(3.40)
	[−1.36]	[3.81]				[−2.60]	[3.08]
2, 5	−.0020	1.3653	.278	.0160	19.57	−.0005	.1555
	(−1.53)	(4.12)				(−2.65)	(3.16)
	[−1.45]	[3.58]				[−2.57]	[2.90]
3, 3	−.0030	1.8689	.297	.0208	17.86	−.0009	.2700
	(−1.66)	(4.17)				(−2.71)	(3.40)
	[−1.57]	[3.62]				[−2.62]	[3.08]
3, 4	−.0023	1.5449	.288	.0176	18.75	−.0007	.1922
	(−1.54)	(4.33)				(−2.66)	(3.43)
	[−1.45]	[3.72]				[−2.58]	[3.10]
3, 5	−.0021	1.3463	.287	.0154	16.68	−.0005	.1462
	(−1.57)	(3.98)				(−2.62)	(3.17)
	[−1.49]	[3.49]				[−2.54]	[2.91]

Table II—*Continued*

Portfolios i, j	a_{0ij}	a_{2ij}	R^2	$S(\hat{u})$	$SR(\hat{u})$	α_{0ij}	α_{2ij}
4, 4	−.0016	1.3010	.276	.0153	19.02	−.0005	.1422
	(−1.34)	(4.53)				(−2.64)	(3.49)
	[−1.26]	[3.84]				[−2.56]	[3.15]
4, 5	−.0015	1.1379	.278	.0133	16.88	−.0004	.1075
	(−1.37)	(4.10)				(−2.63)	(3.23)
	[−1.29]	[3.57]				[−2.55]	[2.95]
5, 5	−.0013	1.0204	.272	.0121	16.93	−.0003	.0886
	(−1.32)	(3.75)				(−2.61)	(3.00)
	[−1.25]	[3.33]				[−2.53]	[2.77]

Table III

Weighted Least Squares Estimates of the Single Index Model for Time-Varying Covariances Among Monthly Returns to Size-Ranked Portfolios, 1927–1986

Equation (9) is estimated using products of excess returns to equally weighted size-ranked portfolios, where R_{1t} is the excess return to the smallest firm portfolio and R_{5t} is the excess return to the portfolio of the largest firms. The predicted variance of the CRSP equally weighted portfolio return $\hat{\sigma}^2_{et}$ is from Table I. R^2 is the coefficient of determination and $S(\hat{u})$ is the residual standard deviation, both in the original units of the data. $SR(\hat{u})$ is the Studentized range of the weighted residuals. These equations are estimated using iterated weighted least squares (WLS), where the squared residuals, $\hat{u}^2_{ijt}$, are regressed against the predicted variance $\hat{\sigma}^2_{et}$ in the Glejser regression (10), and then (9) is reestimated using WLS, and then (10) is estimated using WLS, and so forth. This procedure, recommended by Davidian and Carroll (1987), is repeated three times. t-statistics are in parentheses under the coefficient estimates. t-statistics in brackets correct the WLS standard errors for the fact that the regressor $\hat{\sigma}^2_{et}$ is a function of the predictions from Table I, using the technique of Murphy and Topel (1985).

$$(R_{it} \cdot R_{jt}) = a_{0ij} + a_{2ij}\hat{\sigma}^2_{et} + u_{ijt}, \tag{9}$$

$$\hat{u}^2_{ijt} = \alpha_{0ij} + \alpha_{2ij}\hat{\sigma}^2_{et} + e_{ijt}. \tag{10}$$

Portfolios i, j	a_{0ij}	a_{2ij}	R^2	$S(\hat{u})$	$SR(\hat{u})$
1, 1	−.0040	3.2072	.177	.0615	15.58
	(−5.59)	(9.00)			
	[−3.21]	[5.71]			
1, 2	−.0023	2.2204	.215	.0377	15.02
	(−5.12)	(9.88)			
	[−2.74]	[5.91]			
1, 3	−.0012	1.8638	.237	.0319	12.03
	(−1.64)	(7.74)			
	[−1.26]	[5.35]			
1, 4	−.0017	1.5488	.235	.0261	13.03
	(−2.31)	(7.02)			
	[−1.90]	[5.09]			
1, 5	−.0014	1.2828	.234	.0228	21.39
	(−2.08)	(6.05)			
	[−1.77]	[4.69]			
2, 2	−.0009	1.6221	.260	.0246	16.96
	(−.92)	(6.18)			
	[−.81]	[4.73]			

Table III—*Continued*

Portfolios i, j	a_{0ij}	a_{2ij}	R^2	$S(\hat{u})$	$SR(\hat{u})$
2, 3	−.0003	1.4578	.266	.0222	13.20
	(−.49)	(7.38)			
	[−.39]	[5.21]			
2, 4	−.0001	1.1856	.262	.0186	14.92
	(−.13)	(7.68)			
	[−.09]	[5.31]			
2, 5	−.0026	.7916	.232	.0165	17.36
	(−3.47)	(3.75)			
	[−3.29]	[3.35]			
3, 3	−.0014	1.3366	.270	.0212	15.20
	(−2.13)	(7.15)			
	[−1.78]	[5.13]			
3, 4	−.0003	1.1797	.272	.0178	12.09
	(−.68)	(7.62)			
	[−.52]	[5.29]			
3, 5	.0000	1.0040	.268	.0156	18.58
	(.06)	(6.52)			
	[.05]	[4.88]			
4, 4	.0002	.9918	.267	.0154	15.52
	(.52)	(7.48)			
	[.38]	[5.24]			
4, 5	−.0004	.8803	.267	.0134	12.46
	(−1.38)	(8.76)			
	[−.97]	[5.62]			
5, 5	.0002	.7836	.260	.0122	12.04
	(.46)	(7.07)			
	[.37]	[5.09]			

p. 178) analyzes volatilities for individual firms over the period 1962–1975 and concludes the following:

> As expected, there seems to be a lot of commonality in volatility changes across stocks. A 1% market volatility change typically implies a 1% volatility change for each stock. Well, perhaps the high volatility stocks are somewhat more sensitive to market volatility changes than the low volatility stocks. In general, it seems fair to say that when stock volatilities change they all tend to change in the same direction.

Our results support Black's conclusion that volatilities tend to move in the same direction. As for his proposition that the proportional changes in volatilities are not the same for all firms or portfolios, our estimates $\hat{a}_{211}$ and $\hat{a}_{255}$ show that small (most volatile) firm portfolio variances are four times more sensitive to market volatility changes than large (least volatile) firm portfolio variances.

The Murphy-Topel t-statistics are about 10 percent smaller than the Hansen t-statistics for the slope estimates $\hat{a}_{2ij}$. The Studentized range statistics $SR(\hat{u})$ are large (over 16.8), consistent with heteroskedasticity (and non-normality) in the errors from (9). David, Hartley, and Pearson (1954) propose this statistic as a test for fat-tailed distributions (relative to a normal population) or for heteroskedasticity. Even if excess stock returns R_{it} were normally distributed, products of returns would not be, so it is not surprising that the errors from (9) have large $SR(\hat{u})$ statistics.[3]

The estimates of the Glejser regression (10) in Table II show that the errors from (9) are also heteroskedastic. All the estimates of the slopes $\hat{\alpha}_{2ij}$ are positive and more than 2.7 standard errors from zero, while the estimates of the intercepts $\hat{\alpha}_{0ij}$ are all negative and about 2.5 standard errors from zero.

The WLS estimates in Table III have a pattern similar to the OLS estimates, and the t-statistics are larger. The $SR(\hat{u})$ statistics are smaller for the weighted WLS residuals than for the OLS residuals. The WLS estimates of the slope coefficient $\hat{a}_{2ij}$ are smaller than the OLS estimates in Table II. Since both OLS and WLS estimators are unbiased and consistent when the model specification is correct, these results raise questions about the adequacy of the model specification. Nevertheless, the t-test for the equality of the OLS and WLS estimates of a_{211}, the slope coefficient for the small firm portfolio variance, is only 0.86. Hausman (1978) shows that the variance of the difference between two consistent estimates, one of which is efficient, is the difference in the variances.

The WLS estimates of the intercept $\hat{a}_{0ij}$ in Table III are within two standard errors of 0, except for the covariances involving the small firm portfolio 1 and the covariance between the next smallest firm portfolio 2 with the large firm portfolio 5. This implies that heteroskedasticity is almost proportional, except for the small firm stocks. Thus, the portfolio variances and covariances move in proportion to the aggregate market variance. In general, the results in Tables II and III show that the single factor model in (9) captures much of the heteroskedasticity in monthly stock returns.

Figure 1 compares the predicted standard deviation of the equally weighted market portfolio from Table I, $\hat{\sigma}_{et}$, with the predicted standard deviation of the equally weighted portfolio of the size-ranked portfolios from Table III, $\tilde{\sigma}_{et}$. We computed the variance $\tilde{\sigma}_{et}^2$ as the weighted average of all of the elements of the covariance matrix, $\hat{\sigma}_{ijt}$:

$$\tilde{\sigma}_{et}^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} \omega_i \omega_j \hat{\sigma}_{ijt}, \quad \omega_i = \frac{1}{5}. \tag{11}$$

While there are periods when the plots differ, such as the mid-1930's and mid-1960's, the correspondence between the two predictions is close. The correlation coefficient between $\hat{\sigma}_{et}$ and $\tilde{\sigma}_{et}$ is 0.96. Robert Engle has pointed out that, by allowing for nonproportional heteroskedasticity in (9), we are not imposing that the covariance matrix $\boldsymbol{\Sigma}_t = \{\sigma_{ijt}\}$ is positive definite. Indeed, a few of the values of $\tilde{\sigma}_{et}^2$ are negative during the 1927–1986 sample, so they were set to zero to calculate the standard deviations.

C. The "Market Model" Regression with Time-Varying Betas

The market model regression equation,

$$R_{it} = \alpha_i + \beta_i R_{et} + \epsilon_{it}, \quad t = 1, \cdots, T, \tag{12}$$

[3] For example, for the case of variance terms R_{it}^2, if R_{it} is normally distributed, the square would be distributed proportionally to a χ^2 variable with 1 degree of freedom.

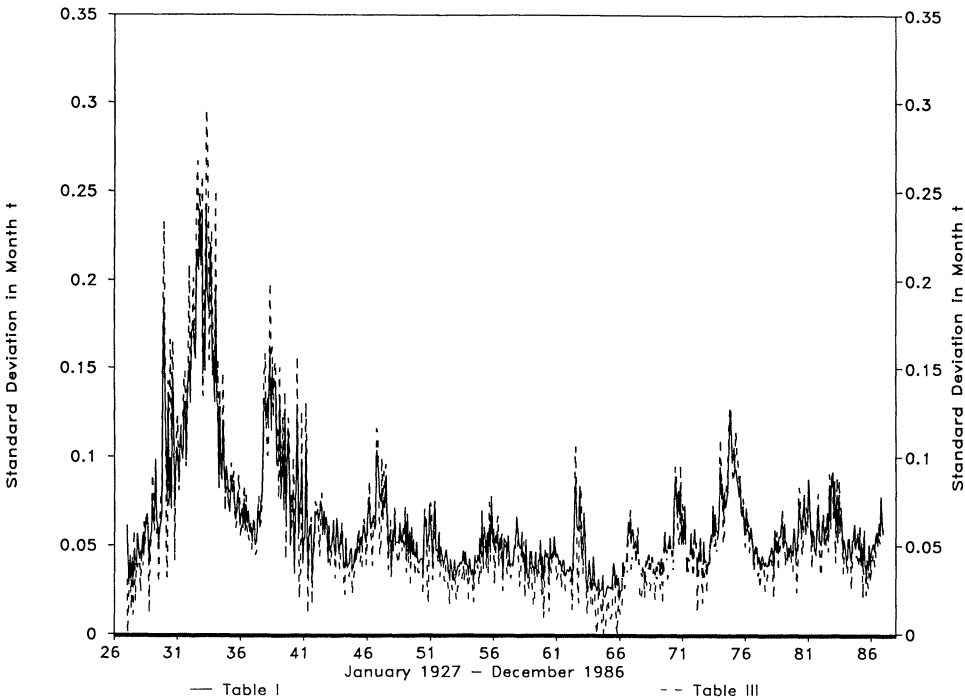

Figure 1. Predictions of the CRSP equally weighted return standard deviation from the regression model in Table I and the equally weighted average of the predictions of conditional covariances of size-ranked portfolio returns from the single index regression model in Table III, 1926–1986. Daily returns to the Standard & Poor's composite portfolio are used to estimate the monthly standard deviation for month t. A regression model using 12 lags of this S&P standard deviation is used to predict the standard deviation of the CRSP equally weighted portfolio return. The solid line represents the fitted values from this regression (in Table I). These fitted values are then used to predict the variances and covariances of returns to five equally weighted size-ranked portfolios of New York Stock Exchange-listed stocks. The equally weighted average of these variances and covariances is an estimate of the variance of the equally weighted portfolio of all stocks. The square root of the equally weighted average of the fitted values for all of the variances and covariances is shown as the broken line (from Table III).

where R_{et} is the excess return to a "market" portfolio of assets, is frequently used in the finance literature. The slope coefficient β_i is a measure of the relative nondiversifiable risk of security i as part of the market portfolio e, $\beta_i = \mathrm{cov}(R_{it}, R_{et})/\sigma_{et}^2$. The "beta coefficient" β_i is a linear combination of the elements of the covariance matrix of returns to securities in the market portfolio:

$$\beta_i = \sum_{i=1}^{N} \omega_j \sigma_{ij} \Bigg/ \left[\sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij} \right], \tag{13}$$

where ω_i represents the weight of security i in portfolio e. Based on the single index model in (2), the beta coefficient for security i in period t is

$$\beta_{it} = \frac{\sum_{j=1}^{N} \omega_j (a_{0ij} + a_{2ij} \sigma_{et}^2)}{\sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j (a_{0ij} + a_{2ij} \sigma_{et}^2)}, \tag{14}$$

which specializes to

$$\beta_{it} = \frac{\bar{a}_{0i}}{\sigma^2_{et}} + \bar{a}_{2i} \tag{15}$$

using the constraint that the weighted average beta coefficient equals 1.[4] The notation $\bar{a}_{mi}$ represents $\sum \omega_j a_{mij}$, for $m = 0, 2$. Thus, if disaggregate heteroskedasticity is nonproportional ($\bar{a}_{0i} \neq 0$), the beta coefficient for portfolio i will vary with the level of aggregate volatility. This possibility can be incorporated into the market model by adding a term $(R_{et}/\hat{\sigma}^2_{et})$, which we refer to as the heteroskedastic market model:

$$R_{it} = \alpha_i + \beta_i R_{et} + \delta_i (R_{et}/\hat{\sigma}^2_{et}) + \epsilon_{it}, \quad t = 1, \cdots, T, \tag{16}$$

so that $\beta_{it} = \beta_i + \delta_i/\hat{\sigma}^2_{et}$, where δ_i measures $\bar{a}_{0i}$.

Table IV contains estimates of the heteroskedastic market model (16), along with estimates of the Glejser regression:

$$(\pi/2)^{1/2} | \hat{\epsilon}_{it} | = \nu_{0i} + \nu_{1i} \hat{\sigma}_{et} + v_{it}. \tag{17}$$

Table IV also contains WLS estimates of (16) using the predicted residual standard deviations from (17) to construct weights. The OLS estimates of (16) are consistent with evidence from Banz (1981) and other papers, where the beta estimate $\hat{\beta}_i$ is monotonically decreasing in firm size, as is the risk-adjusted average return $\hat{\alpha}_i$. The positive $\hat{\alpha}_1$ shows that the portfolio of small firm stocks earns average returns that are larger than the predictions of the Sharpe-Lintner CAPM.[5] The Murphy-Topel t-statistics are not much different from the Hansen t-statistics in this table, because the generated regressor $(R_{et}/\hat{\sigma}^2_{et})$ has relatively small explanatory power compared with R_{et}.

The least squares estimate of δ_i is reliably negative for the small firm portfolio 1, and it is positive for the larger firm portfolios. This pattern implies that the spread between the risk of small and large stocks is larger during periods of high aggregate stock market volatility. For example, during the 1929–1933 period when aggregate stock volatility was highest, the small firm portfolio had a $\hat{\beta}_{1t}$ close to 1.4, and the large firm portfolio had a $\hat{\beta}_{5t}$ of about 0.75. During the mid-1960's, when aggregate volatility was lower, the estimated spread between small and large firm $\hat{\beta}_{it}$'s was close to 0. This is illustrated in Figure 2, which shows $\hat{\beta}_{1t}$ (smallest firms) and $\hat{\beta}_{5t}$ (largest firms) based on the WLS estimates of (16) from 1927 to 1986.

The $SR(\hat{\epsilon})$ statistics in Table IV suggest strong residual heteroskedasticity, and the estimates of the Glejser regression (17) support this interpretation. The WLS estimates of (16) yield similar estimates of α_i and β_i. The $SR(\hat{\epsilon})$ statistics show that the WLS residuals are much less fat-tailed than the OLS residuals

[4] Since the weighted average beta must equal 1.0 for each time t, $\Sigma\, \omega_i \bar{a}_{0i} = 0$, and $\Sigma\, \omega_i \bar{a}_{2i} = 1$. Thus, the denominator of (14) must equal σ^2_{et} by construction.

[5] See Banz (1981) or Black, Jensen, and Scholes (1972) for a discussion of this test. Roll (1977) stresses that this procedure can also be interpreted as a test of whether the "market" portfolio (in this case the NYSE equally weighted portfolio) is mean-variance efficient and the tangency portfolio relative to the risk-free rate. We will refer to this as a test of the CAPM throughout the paper. See Chan and Chen (1988) for a discussion of the distinction between conditional and unconditional tests of the CAPM when the distribution of security returns varies over time.

Table IV

Estimates of the Heteroskedastic Market Model Regression for Monthly Returns to Size-Ranked Portfolios, 1927–1986

Equation (16) is estimated using the excess returns to equally weighted size-ranked portfolios, where R_{1t} is the excess return to the portfolio of smallest firms, R_{5t} is the excess return to the portfolio of the largest firms, and the regressor R_{et} is the excess return to an equally weighted portfolio of NYSE stocks. The predicted variance of the CRSP equally weighted portfolio return $\hat{\sigma}_{et}^2$ is from Table I. The t-statistic under α_i tests whether the average risk-adjusted return to this portfolio equals 0. The t-statistic under β_i tests whether the relative nondiversifiable risk equals 1. The coefficient δ_i represents the time-varying component of the relative nondiversifiable risk, "beta." All of these coefficients are multiplied by 100. The t-statistic under δ_i tests whether relative nondiversifiable risk varies with market volatility. R^2 is the coefficient of determination, $S(\hat{\epsilon})$ is the residual standard deviation, and $SR(\hat{\epsilon})$ is the Studentized range of the residuals (the weighted residuals for WLS). The WLS estimates iterate three times between (16) and (17), as recommended by Davidian and Carroll (1987). For OLS, the t-statistics in parentheses use Hansen's (1982) heteroskedasticity consistent standard errors, with 12 lags of the residuals and the regressors and a damping factor of 0.7. (See the RATS computer manual for details.) t-statistics in brackets correct the Hansen or WLS standard errors for the fact that the regressor $(R_{et}/\hat{\sigma}_{et}^2)$ is a function of the predictions from Table I, using the technique of Murphy and Topel (1985).

$$R_{it} = \alpha_i + \beta_i R_{et} + \delta_i (R_{et}/\hat{\sigma}_{et}^2) + \epsilon_{it}, \tag{16}$$

$$(\pi/2)^{1/2} |\hat{\epsilon}_{it}| = \nu_{0i} + \nu_{1i}\hat{\sigma}_{et} + v_{it}. \tag{17}$$

Portfolio i	α_i	β_i	$100 \cdot \delta_i$	R^2	$S(\hat{\epsilon})$	$SR(\hat{\epsilon})$	ν_{0i}	ν_{1i}
			OLS Estimates					
1	.0017	1.4488	−.0366	.919	.0326	11.14	.0046	.3453
	(1.27)	(16.76)	(−3.69)				(1.73)	(8.06)
	[1.27]	[16.45]	[−3.45]				[1.52]	[6.61]
2	−.0010	1.0804	.0032	.985	.0106	11.05	.0027	.1059
	(−1.88)	(3.86)	(.69)				(3.54)	(8.25)
	[−1.88]	[3.86]	[.69]				[3.13]	[6.82]
3	−.0006	.9931	.0039	.969	.0141	15.84	−.0062	.2498
	(−1.60)	(−.39)	(1.26)				(−1.76)	(3.90)
	[−1.60]	[−.39]	[1.25]				[−1.69]	[3.70]
4	−.0013	.8356	.0172	.935	.0181	18.26	−.0040	.2775
	(−2.07)	(−4.26)	(2.10)				(−1.13)	(4.38)
	[−2.07]	[−4.26]	[2.05]				[−1.08]	[4.09]
5	−.0012	.7099	.0147	.877	.0217	17.41	.0004	.2715
	(−1.13)	(−6.57)	(1.56)				(.14)	(5.37)
	[−1.13]	[−6.56]	[1.54]				[.13]	[4.87]
			WLS Estimates					
1	.0015	1.3829	−.0204	.918	.0328	9.51		
	(1.61)	(13.90)	(−3.70)					
	[1.61]	[13.84]	[−3.47]					
2	−.0007	1.1000	−.0018	.985	.0106	7.87		
	(−2.17)	(11.09)	(−.88)					
	[−2.17]	[11.08]	[−.88]					
3	−.0004	1.0470	−.0041	.967	.0145	26.17		
	(−5.20)	(2.82)	(−3.92)					
	[−5.19]	[2.82]	[−3.59]					
4	−.0008	.8827	.0056	.934	.0183	8.23		
	(−1.99)	(−9.13)	(2.36)					
	[−1.99]	[−9.13]	[2.30]					
5	−.0005	.7328	.0085	.876	.0218	8.93		
	(−.77)	(−15.68)	(2.34)					
	[−.77]	[−15.66]	[2.28]					

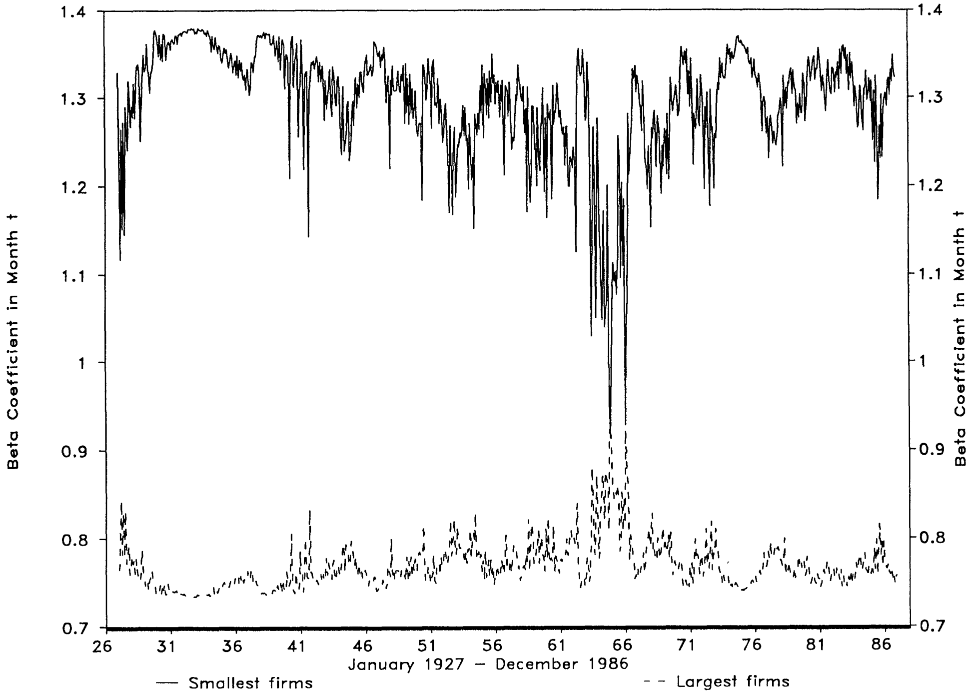

Figure 2. Effects of conditional market volatility on relative risk: Weighted least squares estimates of time-varying beta coefficients for portfolios 1 (small firms) and 5 (large firms) from Table IV, 1927–1986. Returns to equally weighted size-ranked portfolios of New York Stock Exchange-listed stocks are regressed against the equally weighted market return R_{et} and R_{et} divided by the squared predicted standard deviation of the CRSP equally weighted excess market return from the WLS estimates of equation (7b) in Table I, $\hat{\sigma}_{et}^2$

$$R_{it} = \alpha_i + \beta_i R_{et} + \delta_i (R_{et}/\hat{\sigma}_{et}^2) + \epsilon_{it}, \quad t = 1, \cdots, T, \tag{16}$$

The estimate of risk for portfolio i in period t is $\hat{\beta}_{it} = \hat{\beta}_i + \hat{\delta}_i/\hat{\sigma}_{et}^2$. This figure shows the time-varying beta estimates $\hat{\beta}_{it}$ for the small and larger firm portfolios.

(except portfolio 3). The WLS estimates of δ_i increase in firm size, and all but one are more than two standard errors from 0.

Table V contains tests of the CAPM ($\alpha_i = 0$) and tests for constant risk or proportional heteroskedasticity ($\delta_i = 0$) in the heteroskedastic market model (16). Test statistics are created by estimating constrained and unconstrained systems of regressions. The likelihood ratio statistic is proportional to the difference in the logarithms of the determinants of the two residual covariance matrices. We use the Box (1949) correction to the likelihood ratio test to estimate p-values.[6] Two tests are computed for each sample period: (a) a test based on the OLS estimates of (16) and (b) a test based on the WLS estimates of (16), where the fitted values from the Glejser regression (17) estimated for the corresponding sample period are used as weights. In addition, the subperiod tests are summed to calculate an overall test. We compute two versions of the CAPM test, one

[6] We also calculated a number of related test statistics, all of which yielded similar results.

Table V

Cross-Sectional Tests of the Time-Invariant Capital Asset Pricing Model for Equally Weighted Size-Ranked Portfolios, 1927–1986

These are asymptotic χ^2 tests across all five equations in Table IV of the hypothesis that the intercepts in equation (16) α_i equal 0 or that all of the coefficients representing the time-varying component of risk δ_i equal 0. Equation (16) is estimated using the excess returns to equally weighted size-ranked portfolios, where R_{1t} is the excess return to the portfolio of smallest firms, R_{5t} is the excess return to the portfolio of the largest firms, and the regressor R_{et} is the excess return to an equally weighted portfolio of NYSE stocks. The predicted variance of the CRSP equally weighted portfolio return $\hat{\sigma}_{et}^2$ is from Table I. The columns labeled "$\alpha_i = 0 \mid \delta_i = 0$" contain tests of whether the intercept equals 0 when the time-varying component of risk δ_i is constrained to equal 0 (i.e., in the market model regression (12)). This is the conventional Sharpe-Lintner CAPM test. These statistics should be distributed as a χ^2 variable with 5 degrees of freedom. The tests are based on the determinants of the residual covariance matrices under the null and alternative hypotheses (i.e., likelihood ratio tests). The Box (1949) approximation is used to estimate the p-value for the test statistics (in parentheses under the test statistics). WLS estimates use the iterated Glejser estimates from (17) for the subperiod in question. The last row shows the sum of the χ^2 statistics across the six subperiods along with the p-value from a χ^2 distribution with 30 degrees of freedom. We make no correction for the "generated regressors" problem with the tests in the columns labeled "$\alpha_i = 0$" and "$\delta_i = 0$".

$$R_{it} = \alpha_i + \beta_i R_{et} + \delta_i (R_{et}/\hat{\sigma}_{et}^2) + \epsilon_{it}, \tag{16}$$

$$(\pi/2)^{1/2} |\hat{\epsilon}_{it}| = \nu_{0i} + \nu_{1i}\hat{\sigma}_{et} + v_{it}. \tag{17}$$

Sample Period	OLS			Glejser WLS		
	$\alpha_i = 0 \mid \delta_i = 0$	$\alpha_i = 0$	$\delta_i = 0$	$\alpha_i = 0 \mid \delta_i = 0$	$\alpha_i = 0$	$\delta_i = 0$
1/27–12/86	8.75	11.16	41.53	28.28	36.27	23.00
	(.1197)	(.0483)	(.0000)	(.0000)	(.0000)	(.0003)
1/27–12/36	8.99	9.48	2.24	17.85	16.60	4.62
	(.1096)	(.0914)	(.8158)	(.0031)	(.0053)	(.4638)
1/37–12/46	14.39	14.03	12.01	18.32	17.27	7.19
	(.0133)	(.0154)	(.0347)	(.0026)	(.0040)	(.2067)
1/47–12/56	6.32	6.23	4.59	8.73	8.61	4.64
	(.2767)	(.2844)	(.4683)	(.1203)	(.1255)	(.4618)
1/57–12/66	6.06	5.89	5.35	6.75	6.29	7.59
	(.3008)	(.3170)	(.3746)	(.2400)	(.2788)	(.1803)
1/67–12/76	5.98	5.08	30.33	8.10	6.74	36.17
	(.3078)	(.4063)	(.0000)	(.1509)	(.2405)	(.0000)
1/77–12/86	6.36	7.06	5.00	7.18	7.98	5.72
	(.2731)	(.2164)	(.4163)	(.2074)	(.1573)	(.3341)
Sum Across Subperiods	48.09	47.77	59.51	66.93	63.51	65.94
	(.0194)	(.0209)	(.0011)	(.0001)	(.0003)	(.0002)

based on the market model in (12) (labeled $\alpha_i = 0 \mid \delta_i = 0$) and one based on the heteroskedastic market model in (16) (labeled $\alpha_i = 0$).

Using the OLS market model test of the CAPM ($\alpha_i = 0 \mid \delta_i = 0$), the p-value is 0.12 for the 1927–1986 sample period. The CAPM test has p-values larger than 0.25 in most of the ten-year subperiods, but the aggregate test across all six subperiods has a p-value of about 0.02. The OLS heteroskedastic market model tests of the CAPM ($\alpha_i = 0$) yield similar results. The WLS tests, however, provide stronger evidence against the CAPM since all of these tests are larger (the p-values are smaller) than their OLS counterparts. The p-values are less than 0.0004 in the 1927–1986 sample period and across all of the subperiods. Based on the subperiod results, the strongest evidence against the CAPM occurs in the 1927–1936 and 1937–1946 subperiods.

The tests aggregated across the subperiods in the bottom row of Table V allow different parameter values in (16) in each sample period. In contrast, the test for the whole 1927–1986 sample period in the first row imposes constant parameters in (16) for the entire period. If the parameters were truly constant, the full sample test has greater power, since it is not necessary to estimate extraneous parameters. On the other hand, if the parameters in (16) vary over time, the sum of the subperiod tests in the bottom row will have more power to detect departures from the null hypothesis. Since these tests have higher p-values, it is reasonable to conclude that departures from the CAPM are more important in some of the subperiods than in others. Since the tests for each nonoverlapping subperiod are asymptotically χ^2 with 5 degrees of freedom, the sum of these tests should be distributed χ^2 with 30 degrees of freedom.[7]

The results of tests for time-varying risk ($\delta_i = 0$) are similar to the results of tests of the CAPM. For the overall sample period, and summed across all of the subperiods, the p-values for this test are less than 0.002. Nevertheless, the evidence for time-varying betas is concentrated in two subperiods, 1937–1946 and 1967–1976.

Thus, the evidence in Table V for time-varying betas, as represented by the heteroskedastic market model (16), is about as strong as the evidence against the CAPM usually referred to as the "small firm effect". In general, these test statistics are larger using the Glejser WLS estimates of the model. These tests are asymptotically more powerful than the OLS tests, so this result should not be surprising if the null hypothesis is false.

II. The Constant Correlation Single Index Model

As mentioned above, Davidian and Carroll (1987) argue that standard deviation specifications are generally more robust than variance specifications. The effects of extreme returns on the model estimates are smaller when they are not squared as in (9). As shown in (4) and (5), if one is willing to assume that the correlation matrix is constant over time, the time-varying covariance matrix of returns $\mathbf{\Sigma}_t$ can be estimated using the time-varying standard deviations of portfolio returns. The portfolio standard deviations can be estimated using the regression:

$$(\pi/2)^{1/2} \,|\, R_{it} - \hat{\mu}_i \,| = s_{0i} + s_{1i}\hat{\sigma}_{et} + u_{it}, \quad t = 1, \cdots, T, \tag{18}$$

where $|\, R_{it} - \hat{\mu}_i \,|$ is the absolute value of the excess return to portfolio i in period t minus the sample average of excess returns $\hat{\mu}_i$. In (18), s_{0i} represents the part of the portfolio standard deviation that is constant over time, and s_{1i} represents the part that is proportional to the aggregate standard deviation $\hat{\sigma}_{et}$. Based on the arguments of Davidian and Carroll (1987), estimates of (18) in Table VI

[7] See Gibbons and Shanken (1987) for a discussion of aggregation of such test statistics across nonoverlapping sample periods.

Table VI

Estimates of the Constant Correlation Single Index Model for Time-Varying Standard Deviations of Monthly Returns to Size-Ranked Portfolios, 1927–1986

Equation (18) is estimated using estimates of the standard deviations of the excess returns to equally weighted size-ranked portfolios, $(\pi/2)^{1/2}|R_{it} - \hat{\mu}_i|$, where R_{1t} is the excess return to the portfolio of smallest firms, R_{5t} is the excess return to the portfolio of the largest firms, and $\hat{\mu}_i$ is the average excess return for portfolio i in the sample period. The predicted standard deviation of the CRSP equally weighted portfolio return $\hat{\sigma}_{et}$ is from Table I. R^2 is the coefficient of determination, $S(\hat{u})$ is the residual standard deviation, and $SR(\hat{u})$ is the Studentized range of the residuals (the weighted residuals for WLS). The WLS estimates iterate three times between (18) and (19), as recommended by Davidian and Carroll (1987). For OLS, the t-statistics in parentheses use Hansen's (1982) heteroskedasticity consistent standard errors, with 12 lags of the residuals and the regressors and a damping factor of 0.7. (See the RATS computer manual for details.) t-statistics in brackets correct the Hansen or WLS standard errors for the fact that the regressor $\hat{\sigma}_{et}$ is the prediction from Table I, using the technique of Murphy and Topel (1985).

$$(\pi/2)^{1/2}|R_{it} - \hat{\mu}_i| = s_{0i} + s_{1i}\hat{\sigma}_{et} + u_{it}, \tag{18}$$

$$(\pi/2)^{1/2}|\hat{u}_{it}| = \gamma_{0i} + \gamma_{1i}\hat{\sigma}_{et} + v_{it}. \tag{19}$$

Portfolio i	s_{0i}	s_{1i}	R^2	$S(\hat{u})$	$SR(\hat{u})$	γ_{0i}	γ_{1i}
			OLS Estimates				
1	−.0226	1.7020	.246	.1007	13.78	−.0234	1.5418
	(−1.65)	(7.47)				(−1.73)	(6.35)
	[−1.47]	[6.30]				[−1.56]	[5.57]
2	−.0117	1.3040	.286	.0697	11.26	−.0090	1.0356
	(−1.21)	(8.03)				(−1.19)	(7.83)
	[−1.06]	[6.61]				[−1.03]	[6.49]
3	−.0115	1.2213	.291	.0645	11.28	−.0099	.9712
	(−1.10)	(6.59)				(−1.14)	(6.25)
	[−.99]	[5.74]				[−1.03]	[5.50]
4	−.0046	1.0187	.274	.0561	11.51	−.0050	.7946
	(−.59)	(7.40)				(−.72)	(6.19)
	[−.52]	[6.24]				[−.65]	[5.46]
5	−.0028	.8746	.262	.0496	10.67	−.0050	.7211
	(−.34)	(5.76)				(−.68)	(5.24)
	[−.31]	[5.17]				[−.63]	[4.77]
			WLS Estimates				
1	−.0039	1.3747	.237	.1013	7.90		
	(−.66)	(10.68)					
	[−.48]	[7.93]					
2	−.0016	1.1298	.280	.0700	6.18		
	(−.33)	(11.67)					
	[−.24]	[8.27]					
3	.0010	1.0060	.282	.0649	6.35		
	(.24)	(11.39)					
	[.17]	[8.18]					
4	.0026	.8970	.269	.0563	7.06		
	(.67)	(11.35)					
	[.49]	[8.15]					
5	.0043	.7537	.256	.0498	6.83		
	(1.26)	(10.85)					
	[.94]	[7.96]					

should be affected less by the non-normality of the errors than the estimates of (9) in Tables II and III. Table VI also contains estimates of the Glejser regression for the errors:

$$(\pi/2)^{1/2} | \hat{u}_{it} | = \gamma_{0i} + \gamma_{1i} \hat{\sigma}_{et} + e_{it}, \quad t = 1, \cdots, T, \tag{19}$$

where $\gamma_{1i} = 0$ implies that there is no heteroskedasticity in (18), and $\gamma_{1i} > 0$ implies that (18) should be estimated using WLS. Finally, Table VI contains iterated WLS estimates of (18).

The estimates of (18) and (19) in Table VI are similar to the results in Table II. The OLS slope estimates $\hat{s}_{1i}$ are between 1.7 and 0.87, and they are larger for the smaller firm portfolios. The intercept estimates $\hat{s}_{0i}$ are close to zero, and they are positively related to firm size. (The intercepts are most negative for the smallest firm portfolio.) The coefficients of determination R^2 are often larger for the standard deviation specification in (18) than for the variance specification in (9), although the differences are not large. The $SR(\hat{u})$ statistics in Table VI are large for the OLS residuals, but they are smaller than for the comparable models in Table II. This indicates less residual heteroskedasticity or non-normality and suggests that the standard deviation specification in (18) provides more efficient estimates of coefficients and fitted values (predicted conditional standard deviations). Finally, the estimates of $\hat{\gamma}_{1i}$ for the Glejser regression (19) show there is substantial heteroskedasticity in the errors from (18).

The WLS estimates in Table VI confirm the analysis of the OLS estimates. The estimates of the slope coefficient $\hat{s}_{1i}$ are between 1.37 and 0.75, and they are at least eight standard errors above 0. The intercepts $\hat{s}_{0i}$ are small and less than 1.5 standard errors from 0. The difference between the WLS t-statistics and the Murphy-Topel adjusted t-statistics is larger than for the previous tables. The Murphy-Topel statistics are about 30 percent smaller in most cases. The $SR(\hat{u})$ statistics of the WLS residuals are between 7.9 and 6.2, much smaller than either the OLS results or the results for the variance specification in Table III.

Table VII contains tests across the five equations for (18) of the hypothesis that all of the intercepts s_{0i} equal 0. This test for proportional heteroskedasticity is an alternative to the time-varying betas in the heteroskedastic market model ($\delta_i = 0$).[8] The tests are performed for ten-year subperiods and for the overall 1927–1986 sample period, using OLS and WLS techniques. At the bottom of the table is the sum of the subperiod statistics, which should be distributed as a χ^2 variable with 30 degrees of freedom under the null hypothesis. For the overall period, the WLS χ^2 statistic is 5.95, with a p-value of 0.31, which confirms the small t-statistics for $\hat{s}_{0i}$ in Table VI. The largest tests (smallest p-values) in Table VII occur in the last two subperiods, 1967–1976 and 1977–1986. Remember that Table V shows significant time-varying risk in the 1967–1976 subperiod.

Consistent with the analysis of Davidian and Carroll (1987), the estimates in Tables VI and VII, especially the uniformly low Studentized range statistics,

[8] We could have performed similar tests on the regression models for variances and covariances in Tables II and III. The extreme non-normality of the errors, however, would create test statistics that were not close to the hypothesized χ^2 distribution under the null hypothesis, so we did not calculate these tests.

Table VII

Cross-Sectional Tests for Proportional Heteroskedasticity in the Constant Correlation Single Index Model for Size-Ranked Portfolios

These are asymptotic χ^2 tests across all five equations in Table VI of the hypothesis that the intercepts in equation (18) s_{0i} equal 0. If this condition is satisfied, the standard deviations of the size-ranked portfolio excess returns move together in proportion to the standard deviation of the equally weighted market excess return. The statistics should be distributed as a χ^2 variable with five degrees of freedom if the intercepts equal 0. The tests are based on the determinants of the residual covariance matrices under the null and alternative hypotheses (i.e., likelihood ratio tests). The Box (1949) approximation is used to estimate the p-value for the test statistics (in parentheses under the test statistics). WLS estimates use the iterated Glejser estimates from the subperiod in question. The last row shows the sum of the χ^2 statistics across the six subperiods along with the p-value from a χ^2 distribution with 30 degrees of freedom. We make no correction for the "generated regressors" problem. Equation (18) is estimated using estimates of the standard deviations of the excess returns to equally weighted size-ranked portfolios, $(\pi/2)^{1/2}|R_{it} - \hat{\mu}_i|$, where R_{1t} is the excess return to the portfolio of smallest firms, R_{5t} is the excess return to the portfolio of the largest firms, and $\hat{\mu}_i$ is the average excess return for portfolio i in the sample period. The predicted standard deviation of the CRSP equally weighted portfolio return $\hat{\sigma}_{et}$ is from Table I.

$$(\pi/2)^{1/2}|R_{it} - \hat{\mu}_i| = s_{0i} + s_{1i}\hat{\sigma}_{et} + u_{it}, \tag{18}$$

$$(\pi/2)^{1/2}|\hat{u}_{it}| = \gamma_{0i} + \gamma_{1i}\hat{\sigma}_{et} + v_{it}. \tag{19}$$

Sample Period	OLS	Glejser WLS
1/27–12/86	25.52	5.95
	(.0001)	(.3114)
1/27–12/36	1.59	8.88
	(.9030)	(.1139)
1/37–12/46	14.81	7.24
	(.0112)	(.2033)
1/47–12/56	7.43	5.73
	(.1907)	(.3333)
1/57–12/66	3.06	2.87
	(.6915)	(.7197)
1/67–12/76	14.18	16.84
	(.0145)	(.0048)
1/77–12/86	13.73	26.91
	(.0174)	(.0001)
Sum Across	54.79	68.47
Subperiods	(.0038)	(.0001)

suggest that the standard deviation specification in (18) may be preferred to the variance specification in (9). Perhaps the main reason for this difference is that the square root transformation in (18) produces errors that are less positively skewed and less affected by large observations.

III. Comparison with Multivariate GARCH Models

Bollerslev, Engle, and Wooldridge (1988), Engle (1988), Baillie and Bollerslev (1987), Bollerslev (1987), and Ng (1988), among others, have proposed using multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models to represent a set of time series whose variances and covariances change over time. In particular, the time-varying covariance matrix can be expressed as a multivariate GARCH (p, q) process:

$$\mathbf{\Sigma}_t = \mathbf{C} + \sum_{i=1}^{p} \mathbf{A}_i \epsilon_{t-i} \epsilon'_{t-i} + \sum_{j=1}^{q} \mathbf{B}_j \mathbf{\Sigma}_{t-j}, \tag{20}$$

where $\mathbf{C}$ represents the time-invariant component of the covariance matrix, and the coefficient matrices $\mathbf{A}_i$ and $\mathbf{B}_j$ determine the persistence of the conditional heteroskedasticity. Engle (1988) discusses a number of alternative parameterizations of the multivariate GARCH model that ensure that the covariance matrix $\mathbf{\Sigma}_t$ is positive definite. Given a model for the conditional mean return (which we treat as a constant for a given security) and the assumption that the conditional distribution of returns is multivariate normal, a nonlinear optimization algorithm can be used to construct maximum likelihood estimates of the parameters and the time-varying covariance matrices.

Table VIII contains estimates of univariate GARCH (1, 1) models for each of the five size-ranked portfolios and for the equally weighted market portfolio:

$$R_{it} = \alpha_i + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_{it}^2), \tag{21}$$

$$\sigma_{it}^2 = c_i + a_{i1}\epsilon_{it-1}^2 + b_{i1}\sigma_{it-1}^2, \quad i = 1, \cdots, 5; \quad t = 1, \cdots, T. \tag{22}$$

These GARCH models were chosen because they seem to approximate the time-varying behavior of the conditional variances. For example, French, Schwert,

Table VIII

Estimates of the Univariate GARCH Models for Monthly Stock Returns to Size-Ranked Portfolios, 1927–1986

These Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, which are special cases of equation (20), are estimated using the excess returns to equally weighted size-ranked portfolios, where R_{1t} is the excess return to the portfolio of smallest firms and R_{5t} is the excess return to the portfolio of the largest firms. The last row shows estimates for R_{et}, the excess return to the CRSP equally weighted portfolio of NYSE stocks. The asymptotic t-statistics are in parentheses under the coefficient estimates. The last column contains the value of the log-likelihood function for these parameter estimates, ln L.

$$R_{it} = \alpha_i + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_{it}^2), \tag{21}$$

$$\sigma_{it}^2 = c_i + a_{i1}\epsilon_{it-1}^2 + b_{i1}\sigma_{it-1}^2, \quad i = 1, \cdots, 5; \quad t = 1, \cdots, T. \tag{22}$$

Portfolio i	α_i	$100 \cdot c_i$	a_{i1}	b_{i1}	ln L
1	.0156	.0080	.1808	.8358	806.15
	(7.89)	(2.41)	(8.14)	(50.27)	
2	.0134	.0118	.1522	.8352	947.26
	(6.96)	(3.00)	(5.91)	(33.50)	
3	.0132	.0106	.1517	.8330	1011.91
	(7.14)	(3.23)	(5.50)	(32.66)	
4	.0122	.0102	.1550	.8262	1079.73
	(6.89)	(3.41)	(5.73)	(34.11)	
5	.0115	.0079	.1538	.8271	1169.67
	(7.12)	(3.08)	(6.25)	(38.65)	
R_e	.0132	.0099	.1619	.8270	1022.24
	(7.23)	(3.00)	(5.91)	(32.57)	

and Stambaugh (1987) find that the GARCH (1, 1) model works well for monthly stock returns. The parameter estimates show that the size-ranked portfolios have very similar properties. The means are a monotonically declining function of firm size, but the GARCH parameters are virtually identical for the different portfolios. Also, there seems to be persistence in the variances of the stock returns since the sum $(a_{i1} + b_{i1})$ is close to unity. Engle (1988) refers to this as an integrated GARCH process. These estimates suggest that the single index model of heteroskedasticity in Sections I and II provides a good description of the data. The conditional volatilities from (22) are highly correlated across portfolios. The lowest correlation among the five portfolios is 0.91, and all adjacent portfolio volatilities, such as σ_{1t} and σ_{2t}, have correlations of at least 0.98.

Table IX contains estimates of (20) for the set of five size-ranked portfolios, where the model is constrained using the assumption that the correlation matrix of returns is constant over time:

$$\sigma_{ijt} = \rho_{ij}\sigma_{it}\sigma_{jt}, \quad i = 1, \cdots, 5; \quad j = 1, \cdots, i; \quad t = 1, \cdots, T. \tag{23}$$

Even though the GARCH parameters are similar for the different portfolios, the equality constraint can be rejected at conventional significance levels. (The likelihood ratio statistic is 36.06, which should be distributed as χ^2 with 8 degrees of freedom in large samples.)

We tried several specifications of the multivariate GARCH model (20) and

Table IX

Estimates of the Multivariate Constant Correlation GARCH Model for Monthly Stock Returns to Size-Ranked Portfolios, 1927–1986

This multivariate GARCH model, which is a special case of equation (20), is estimated using the excess returns to equally weighted size-ranked portfolios, where R_{1t} is the excess return to the portfolio of smallest firms and R_{5t} is the excess return to the portfolio of the largest firms. The conditional covariances are assumed to be proportional to the product of the conditional standard deviations as in (23), where ρ_{ij} is the unconditional correlation of the errors from (21). The asymptotic t-statistics are in parentheses under the coefficient estimates. The last column contains the value of the log-likelihood function for these parameter estimates, ln L.

$$R_{it} = \alpha_i + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_{it}^2), \tag{21}$$

$$\sigma_{it}^2 = c_i + a_{i1}\epsilon_{it-1}^2 + b_{i1}\sigma_{it-1}^2, \quad i = 1, \cdots, 5; \quad t = 1, \cdots, T, \tag{22}$$

$$\sigma_{ijt} = \rho_{ij}\sigma_{it}\sigma_{jt}. \tag{23}$$

Portfolio i	α_i	$100 \cdot c_i$	a_{i1}	b_{i1}	ln L
1	.0138	.0117	.1026	.8907	12033.7
	(9.60)	(7.33)	(16.56)	(178.00)	
2	.0109	.0154	.0860	.8888	
	(12.29)	(13.36)	(18.72)	(186.64)	
3	.0108	.0186	.0914	.8721	
	(12.62)	(16.24)	(16.15)	(141.03)	
4	.0098	.0190	.0906	.8648	
	(11.26)	(13.48)	(14.29)	(112.16)	
5	.0091	.0161	.0876	.8639	
	(10.82)	(9.38)	(13.51)	(92.38)	

could find none that obviously dominated the constant correlation specification in Table IX. What is of most interest from the perspective of this paper is to compare the time series behavior of the conditional variances from the GARCH models and from the single index regression models in Sections I and II.

Figure 3 plots the conditional standard deviation of the equally weighted market portfolio return $\hat{\sigma}_{et}$ from the WLS regression in Table I and from the univariate GARCH model in Table VIII. It is clear from this plot that the two methods of calculating the conditional standard deviation of market returns yield similar results. (The correlation between them is 0.99.)

Figure 4 plots the conditional standard deviation of the equally weighted market portfolio return from the univariate and multivariate GARCH models in Tables VIII and IX. To calculate the conditional variance of the equally weighted market return from the multivariate GARCH model, we compute the weighted

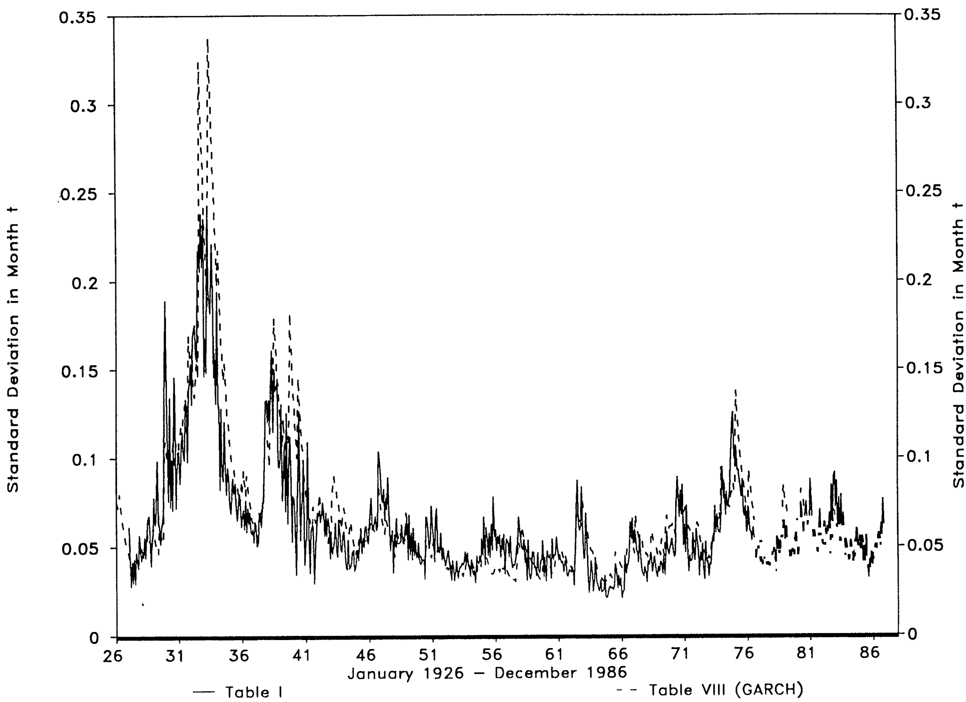

Figure 3. Predictions of the CRSP equally weighted return standard deviation from the regression model in Table I and from the univariate GARCH model in Table VIII, 1926–1986. Daily returns to the Standard & Poor's composite portfolio are used to estimate the monthly standard deviation for month t. A regression model using 12 lags of this S&P standard deviation is used to predict the standard deviation of the CRSP equally weighted portfolio return. The solid line represents the fitted values from this regression (in Table I). A univariate generalized autoregressive conditional heteroskedasticity GARCH (1, 1) model is also estimated for the CRSP equally weighted portfolio return,

$$R_{it} = \alpha_i + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_{it}^2), \tag{21}$$

$$\sigma_{it}^2 = c_i + a_{i1}\epsilon_{it-1}^2 + b_{i1}\sigma_{it-1}^2, \quad i = 1, \cdots, 5; \quad t = 1, \cdots, T. \tag{22}$$

The predicted standard deviations from this model are shown as the broken line (from Table VIII).

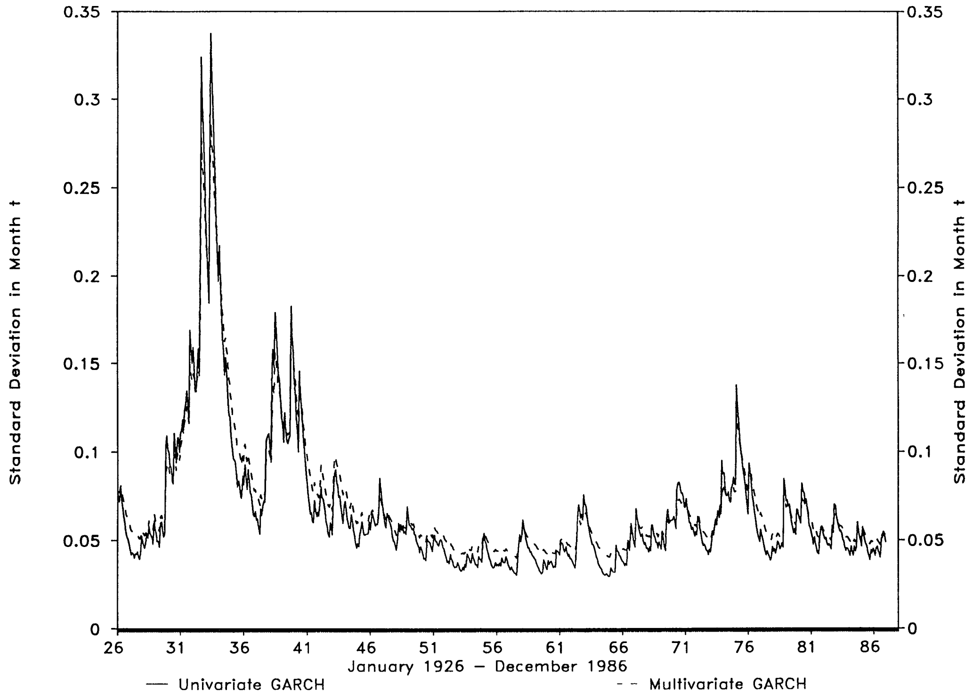

Figure 4. Predictions of the CRSP equally weighted return standard deviation from the univariate GARCH model in Table VIII and the equally weighted average of the predictions of conditional covariances of size-ranked portfolio returns from the multivariate GARCH model in Table IX, 1926–1986. A univariate generalized autoregressive conditional heteroskedasticity GARCH (1, 1) model is estimated for the CRSP equally weighted portfolio return,

$$R_{it} = \alpha_i + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_{it}^2), \tag{21}$$

$$\sigma_{it}^2 = c_i + a_{i1}\epsilon_{it-1}^2 + b_{i1}\sigma_{it-1}^2, \quad i = 1, \cdots, 5; \quad t = 1, \cdots, T. \tag{22}$$

The predicted standard deviations from this model are shown as the solid line (from Table VIII). A multivariate GARCH (1, 1) model is estimated for the five size-ranked portfolio returns assuming constant correlation ρ_{ij},

$$\sigma_{ijt} = \rho_{ij}\sigma_{it}\sigma_{jt}, \quad i = 1, \cdots, 5; \quad j = 1, \cdots, i; \quad t = 1, \cdots, T. \tag{23}$$

Then, an equally weighted average of the covariances σ_{ijt} is used to estimate the equally weighted market variance. The square root of this average is shown as the broken line.

average of all of the elements of the covariance matrix, $\hat{\sigma}_{ijt}$, in (11). As with the plots from the single index model in Figure 1, the plots in Figure 4 show that there is much comovement in the volatilities of the size-ranked portfolio returns.

Thus, the evidence from the multivariate GARCH model supports the conclusions reached from the regression models in Sections I and II. There is a common source of time-varying volatility across disaggregate stock portfolios.

Since the first draft of this paper was written, we have seen a paper by Engle, Ng, and Rothschild (1989) that also examines the commonality of volatility shifts for size-ranked portfolios. They use a Factor-ARCH model and conclude that one dynamic factor (an aggregate portfolio with larger weights for larger firms) is adequate for modeling conditional variances. Thus, their results reinforce our conclusion that single index models, like those presented here, are useful for modeling volatilities of disaggregate portfolios. The recent paper by Merville and Pieptea (1989) documents common movements in the implied standard deviations derived from options prices for several NYSE stocks.

IV. Conclusions

This paper shows that heteroskedasticity in stock returns is a pervasive phenomenon. Using five portfolios of stocks sorted by firm size, we show that there is a common "market" factor in the heteroskedasticity of monthly stock returns. We use daily returns to the Standard & Poor's composite portfolio to measure aggregate monthly stock volatility. The volatility of monthly returns to the size portfolios is highly related to autoregressive predictions of this market volatility factor.

Much prior research views heteroskedasticity as a purely statistical problem, as a potentially confounding factor in estimating the market model. Martin and Klemkosky (1975), Bey and Pinches (1980), and Barone-Adesi and Talwar (1983) conclude that heteroskedasticity is not a problem in studies of security returns. The first two studies examine the market model (12), while Barone-Adesi and Talwar (1983) use the "quadratic market model" proposed by Kraus and Litzenberger (1976), which adds the squared market return to (12). They then regress the absolute errors from this model on the market return (not its square or absolute value). Because the market return is a poor estimate of market volatility, it is not surprising that these authors reached opposite conclusions to a similar question. It is also not surprising that MacDonald and Morris (1983) and Giaccotto and Ali (1982) find stronger evidence against homoskedasticity when they relate error variances to squared market returns, which are better estimates of market volatility.

To a first approximation, disaggregate heteroskedasticity is proportional to the market factor. There is weak evidence that the heteroskedasticity of returns to the small firm portfolio is nonproportional, which implies that relative risk (beta) changes over time for this portfolio.

We show how tests of the capital asset pricing model are affected by a simple weighted least squares heteroskedasticity correction. In general, the evidence that small firms' stocks earn higher returns than predicted by the CAPM is stronger when the WLS tests are used.

Future research will provide more detailed characterizations of the heteroskedasticity of stock returns. Because there are predictable movements in stock volatility, however, many types of tests should take heteroskedasticity into account. For example, studies of the distributional properties of stock returns should incorporate predictable heteroskedasticity. As shown in this paper, failure to account for predictable heteroskedasticity can lead to the misleading conclusion that the conditional distribution of security returns is much more fat-tailed than a normal distribution. Studies of time-varying expected returns or "mean reversion" in stock returns (e.g., Keim and Stambaugh (1986), French, Schwert, and Stambaugh (1987), Fama and French (1988), or Poterba and Summers (1988)) should correct for the effects of predictable heteroskedasticity. In fact, in the Poterba and Summers (1988) paper the evidence for long-term negative autocorrelation in stock returns is much weaker when heteroskedasticity is taken into account.

Studies of contingent claims pricing with time-varying volatility (e.g., Johnson and Shanno (1987) and Wiggins (1987)) and studies of whether stock volatility changes in association with a particular type of event (e.g., Ohlson and Penman (1985), Skinner (1989), or Tracy (1987)) could take advantage of the simple single index structure proposed here. This structure allows researchers to estimate the conditional variances and covariances of individual stocks or portfolios given conditional market variances. Thus, the large body of research on the prediction of market-wide volatility (including French, Schwert, and Stambaugh (1987), Nelson (1990), Pagan and Schwert (1990), and Schwert (1989, 1990a)) can be brought to bear on the prediction of firm and portfolio volatility.

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