Journal of Financial Economics 4 (1977) 51–78. © North-Holland Publishing Company

STOCK EXCHANGE SEATS AS CAPITAL ASSETS

G. William SCHWERT*

University of Rochester, Rochester, NY 14627, USA

Received February 1976, revised version received May 1976

Stock exchange seats are important assets for securities brokers since they provide access to centralized secondary trading markets for corporate securities at a reduced cost. This paper provides empirical evidence on the dynamic behavior of monthly New York and American Stock Exchange seat prices over the 1926–72 period. Specifically, evidence is presented which: (a) is consistent with a multiplicative random walk model for seat prices; (b) indicates that unexpected changes in the prices of exchange-listed stocks or in the volume of shares traded on the exchange are important new information about the value of seats in each month; and (c) indicates that the market for seats is efficient in assimilating new information and quickly incorporates new information into the prices of seats. In addition, we examine the effect of the infrequent trading of seats on the statistical properties of the models.

1. Introduction

In recent years there has been abundant empirical research into the time series and cross-sectional properties of the prices of financial assets. In particular, the prices of New York Stock Exchange-listed common stocks have been analyzed extensively. This paper will present evidence concerning the time series properties of the prices of other assets which are intrinsically related to American capital markets: stock exchange 'seats' or memberships.

The prices of stock exchange seats are often connected with the health and activity of the Wall Street community. As trading activity and stock prices rise, seat prices rise to reflect prosperity in the brokerage industry. Although the total market value of stock exchange seats is only a small fraction of the value of exchange-listed common stocks, the prices of stock exchange seats are of interest because they represent an important indicator of the profitability of the brokerage industry which provides services to the capital markets.

Section 2 of this paper presents a brief description of the market for New York Stock Exchange (NYSE) seats and some historical data on the supply of NYSE seats. Section 3 describes the univariate time series properties of NYSE and

*I would like to thank all of the people who have commented on earlier drafts of this paper, including G. Benston, F. Black, E. Fama, N. Gonedes, M. Jensen, J. Long, M. Miller, C. Nelson, H. Roberts, M. Scholes, and J. Williams, but I retain responsibility for all remaining errors. The University of Chicago Graduate School of Business provided support for this research.

American Stock Exchange (ASE) monthly seat prices from 1926–72. Section 4 contains a simple model for the determination of seat prices as a function of expectations of future share trading volume and future prices of exchange-listed stocks. If the market for seats is efficient in its use of information, unexpected changes in seat prices will only be related to the new information which becomes available each period, perhaps in the form of unexpected changes in share volume and stock prices. This model is estimated using autoregressive-integrated-moving average (ARIMA) time series models to separate percent changes in share volume into expected and unexpected parts. Section 5 investigates the 'Fisher effect' (due to the inactive trading of seats) in the regression models of seat pricing using the transaction-by-transaction series of NYSE seat prices over the 1952–72 period. The 'Fisher effect' (after Professor Lawrence Fisher) refers to the spurious lead and lag relationships between variables which can occur if the variables are contemporaneously correlated but they are not always measured at the same point in time. Section 6 investigates the relationship between changes in seat prices and the returns to the stocks of publicly-owned NYSE member firms over the 1971–75 period. Although the data are limited by the fact that NYSE member firms have only recently been allowed to sell their stock to the public, this analysis indicates that seat prices are highly related to the overall profitability of these major brokerage firms as reflected in their stocks' prices. Section 7 contains brief concluding remarks and suggestions for future research.

2. The market for stock exchange seats

A NYSE seat gives its owner access to the trading floor of the Exchange at a reduced price. A seat owner may participate in any of four categories of activity on the Exchange floor: (1) *specialists*, hold inventories and deal in specific NYSE-listed securities; (2) *commission brokers*, handle the transactions of non-members which are brought to the NYSE through the offices of their brokerage firms; (3) *floor brokers*, handle the trades of other members for a floor brokerage fee (which is less than the commission charged to non-members); and (4) *floor traders*, trade for their personal accounts at reduced transactions costs.[1] The ability to trade at reduced costs makes the seats valuable to securities brokers and dealers who handle large numbers of transactions. If the NYSE were to dissolve itself, seat-holders would receive equal portions of the net proceeds from the liquidation of the assets of the Exchange. In this sense seats are analogous to equity in the NYSE. Most other American stock exchanges are organized in a similar fashion, so only the NYSE is described specifically.

The supply of NYSE seats has remained relatively constant over time. In 1869, three major securities exchanges merged to form the NYSE as it is now

[1] See Leffler and Farwell (1963, pp. 105–126), for a more detailed description of the history and institutional details of the market for NYSE seats.

known and the 1,060 seats were first considered to be saleable property.[2] In 1879, 40 new seats were sold at $13,000 each to finance a new building for the Exchange. In 1929 a 25 percent 'seat dividend' was issued to all seat-owners. All of these rights were exercised by March 1932, bringing the total number of seats to 1,375. In March 1953, the NYSE authorized a plan to retire some seats and nine seats were repurchased in the market for a total of $336,000. The current number of NYSE seats is still 1,366. The number of seats on the ASE has also remained reasonably constant since its initial organization in 1921.

The market for NYSE seats is an anonymous auction market which is operated by the Secretary of the NYSE. Each time a new bid or ask price is brought to the market all interested participants are informed in an effort to consummate a transaction. While private transfers are permissible, only the prices derived from public trades are reported. There are no direct out-of-pocket transactions costs beyond the costs of applying for admission to the NYSE involved in the market for NYSE seats.

Thus, stock exchange seats are assets which have many properties analogous to NYSE-listed common stocks: (1) they represent residual ownership of the assets of the Exchange; (2) their supply has been relatively constant over time; and (3) they are traded in an auction market which involves relatively small transactions costs. However, there are characteristics of NYSE seats which differ from those of common stocks: (1) the 'dividend' return on seats is a function of its use on the NYSE floor (an inactive seat-owner would receive no cash flow which is attributable to the seat); and (2) seats can only be sold to individuals, not groups or corporations, and they may not be loaned or leased to others, so seats are not divisible into smaller parts (although the ASE and other exchanges have other classes of membership which may serve this purpose). Based on these similarities and differences it is difficult to speculate whether prices of stock exchange seats should behave like the prices of NYSE-listed common stocks or other financial assets. The subsequent empirical evidence sheds light on this question.

3. Random walk model for seat prices

There is a substantial literature which shows that the prices of NYSE-listed common stocks may follow a multiplicative random walk [cf. Cootner (1964)] and that the marginal distribution of the returns on common stocks is slightly more fat-tailed than a normal distribution [cf. Fama (1965a)]. The time series properties of the prices of stock exchange seats and the marginal distribution of the approximate percent changes in seat prices support very similar conclusions. The evidence presented below is consistent with the joint hypothesis that the market for seats is efficient in its use of information and that the expected percent change in seat prices is constant through time.

[2]Doede (1967, pp. 5–20), provides a detailed history of the NYSE from its beginning in 1792.

3.1. Time series properties

The data for monthly NYSE and ASE seat prices were obtained as the last trade in the month from the *Bank and Quotation Record* for the 1926–72 period. There are some statistical problems caused by the fact that seats are not traded on the last day of each month, but these will be examined in section 5. For the purposes of this analysis we examine the natural logarithm of seat prices and the first difference of the log of seat prices, $r_t = \ln(PS_t/PS_{t-1})$, the continuously compounded rate of change in seat prices.

As Fama (1970, 1976) notes, the random walk model is a special case of the 'weak form' of the efficient markets hypothesis since it also assumes that expected returns are constant over time. The autocorrelation functions (ACF's) of the logs of NYSE and ASE seat prices and the rates of change in these prices over the 1926–45 and 1946–72 subperiods in table 1 are consistent with the multiplicative random walk model. The ACF's for the logarithms of NYSE and ASE seat prices are very close to unity at all twelve lags which is characteristic of non-stationary processes like the random walk model. The ACF's for rates of change in NYSE seat prices, r_t, and ASE seat prices, r_{at}, are very small at all lags and the joint test of all twelve lags, $Q(12)$, is not significant based upon its asymptotic χ^2_{12} distribution [Box and Pierce (1970)], except for the ACF of r_{at} for the 1946–72 period. This evidence indicates that the model

$$\ln \tilde{PS}_t = \ln PS_{t-1} + \alpha + \tilde{\varepsilon}_t \tag{1}$$

adequately describes the data, where α is the drift parameter for the random walk and $\tilde{\varepsilon}_t$ is a serially uncorrelated random variable with a mean of zero [tildes (˜) indicate random variables].

The plot of r_t in fig. 1 illustrates the heteroscedasticity which necessitates the use of the 1926–45 and 1946–72 subperiods for the analysis of the ACF's. This visual evidence that the dispersion in $\tilde{r}_t$ is greater in the 1926–45 period than the 1946–72 period[3] is confirmed by the estimates of the standard deviation of $\tilde{r}_t$ in the subperiods in table 2. Similar conclusions can be drawn from data for $\tilde{r}_{at}$. Analysis of the ACF's of r_t and r_{at} over shorter subperiods corroborated the results from the 1926–45 and 1946–72 subperiods.

Thus, there is evidence that the monthly changes in the logs of NYSE and ASE seat prices are serially uncorrelated which implies a multiplicative random walk model for the prices of stock exchange seats. This evidence is similar to the results of the well-known empirical analyses of the behavior of NYSE common stock prices.

3.2. Marginal distribution of seat price changes

As with the distribution of common stock returns, the frequency distributions

[3]A similar decrease in dispersion of common stock returns has been analyzed by Officer (1973) and others.

Table 1

Autocorrelations of seat prices.

	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	$\hat{\rho}_7$	$\hat{\rho}_8$	$\hat{\rho}_9$	$\hat{\rho}_{10}$	$\hat{\rho}_{11}$	$\hat{\rho}_{12}$	$S(\hat{\rho})$	$Q(12)$
1926–45														
$\ln PS_t$	0.98	0.97	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.87	0.85	0.83	0.06	2470
$\ln PS_{at}$	0.99	0.97	0.95	0.94	0.93	0.91	0.90	0.88	0.86	0.84	0.82	0.80	0.06	2410
r_t	0.04	−0.01	−0.12	−0.14	−0.02	0.01	0.11	0.05	0.04	0.13	0.07	−0.04	0.06	19.2
r_{at}	0.02	0.04	−0.09	−0.02	0.10	0.03	0.07	−0.05	0.04	0.04	0.07	0.04	0.06	9.9
1946–72														
$\ln PS_t$	0.99	0.98	0.97	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.06	3540
$\ln PS_{at}$	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.91	0.90	0.89	0.06	3510
r_t	−0.07	−0.08	−0.07	0.06	0.08	−0.03	0.01	−0.11	0.05	−0.04	0.05	−0.01	0.06	14.6
r_{at}	−0.10	0.05	0.05	−0.07	0.08	−0.10	0.03	0.02	−0.13	0.08	−0.14	0.11	0.06	30.2

$S(\hat{\rho}) = n^{-\frac{1}{2}}$ is the asymptotic standard error of $\hat{\rho}_1$ under the null hypothesis that $\rho_1 = 0$.
$Q(12) = n \cdot \Sigma_{i=1}^{12} \hat{\rho}_i^2$, is the Box–Pierce statistic for 12 lags.

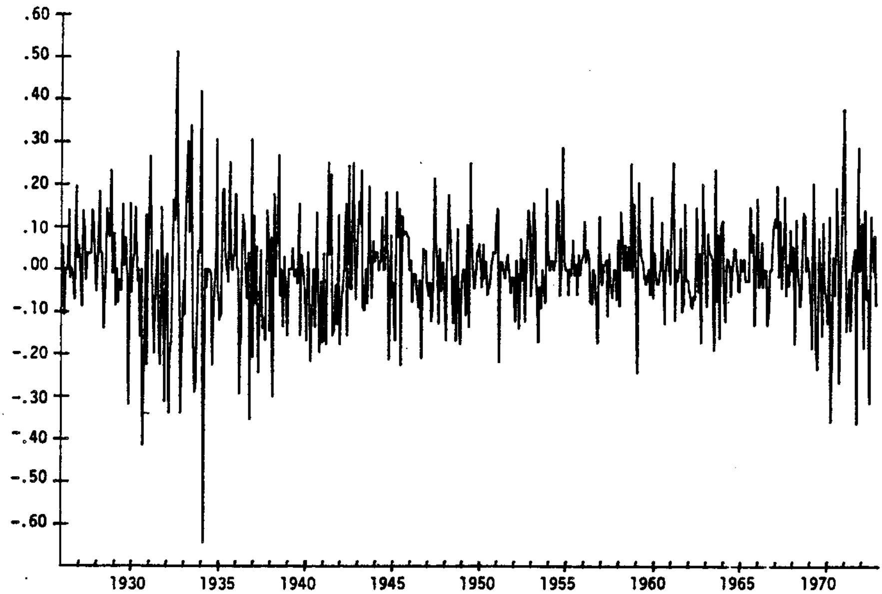

Fig. 1. Monthly rates of change of NYSE seat prices, 1926–72.

Table 2

Summary statistics of r_t and r_{mt}.

	Mean, $\bar{x}$	Standard deviation, $\hat{S}$	Skewness	Kurtosis	Studentized range, S.R.
1926–45					
r_t	−0.00069	0.14760	−0.273	4.60	7.81[a]
r_{mt}	0.00000	0.23132	−0.345	6.55	7.65[a]
1946–72					
r_t	0.00197	0.10176	−0.023	4.41	7.24[a]
r_{mt}	0.00298	0.13626	−0.176	7.37	8.70[a]
1926–72					
r_t	0.00084	0.12325	−0.218	5.21	9.35[a]
r_{mt}	0.00171	0.18267	−0.352	8.48	9.68[a]

[a]S.R. exceeds the 0.99 fractile of the sampling distribution when sampling from a normal population. Sample skewness $= 1/n \sum_{t=1}^{n} [(x_t - \bar{x})/\hat{S}]^3$, sample kurtosis $= 1/n \sum_{t=1}^{n} [(x_t - \bar{x})/\hat{S}]^4$, S.R. $= [(x_{(n)} - x_{(1)})/\hat{S}]$, where $x_{(n)}$ and $x_{(1)}$ are the largest and smallest sample order statistics.

of r_t and r_{at} are reasonably symmetric, but they seem to be slightly leptokurtic. That is, the probability of observing extreme values is too large in comparison with a normal distribution. The sample kurtosis statistics are much larger than the value of 3 which would be expected from a normal distribution and the studentized range statistics [David, Hartley and Pearson (1954)] are also too large. The first four sample moments and the S.R. statistics for r_t and r_{at} are listed in table 2 for the 1926–45 and 1946–72 subperiods and the entire 1926–72 period.

There are at least two possible explanations for the leptokurtosis in the frequency distributions for r_t and r_{at}. Either the marginal distributions of $\tilde{r}_t$ and $\tilde{r}_{at}$ are fat-tailed non-normal distributions, or the parameters of the hypothetical normal distributions for $\tilde{r}_t$ and $\tilde{r}_{at}$ may not be constant within the sample periods. Blattberg and Gonedes (1974) derive the Student-t and Stable Paretian marginal distributions for common stock returns under the assumption that returns are generated by a normal subordinated stochastic process with a directing process which determines the variance of the normal distribution in each period. Thus, these two competing explanations for leptokurtosis may both be attributed to changes in the variance of the hypothetical normal distribution within the sample period, although the subordinated stochastic process model implies random values of the variance parameter which are drawn from a stationary distribution each period.

The estimates of the standard deviations of $\tilde{r}_t$ and $\tilde{r}_{at}$ are much larger in the 1926–45 period than the 1946–72 period, and this is reflected in the large estimates of kurtosis and S.R. for the entire 1926–72 period. However, even within the subperiods there seems to be some evidence of changes in variance. When the sample moments were computed over shorter subperiods of 8 years in length the estimates of the standard deviation differed across periods and the estimates of the kurtosis and S.R. statistics in each subperiod were more consistent with the hypothetical stationary normal distribution for $\tilde{r}_t$ and $\tilde{r}_{at}$. Thus, it appears that the variability of seat price changes may change gradually over time with a pronounced difference between the 1926–45 and post–1945 levels of variability. This evidence does not suggest that the variance is drawn from a serially independent stationary directing process, as suggested by the subordinated stochastic process model, since that would imply that analysis of shorter sample periods would not alleviate the leptokurtosis problem.

Thus, the univariate time series properties of stock exchange seat prices are analogous to the familiar properties of common stock prices: (1) changes in the logs of seat prices are serially uncorrelated, (2) the frequency distribution of log price changes is fat-tailed relative to a normal distribution, at least over long sample periods, and (3) the variability of log price changes was much higher before 1945 than it has been since 1945. This evidence is consistent with a multiplicative random walk for seat prices with a normally distributed increment, although the variance of the increment may change over time.

4. Time series model of seat price determination

The prices of stock exchange seats, PS_t, are related to expected future profits which accrue to seat-holders, ${}_t\pi^*_{t+k}$, by the relationship

$$PS_t = \sum_{k=1}^{\infty} \frac{{}_t\pi^*_{t+k}}{[1+E(\tilde{R}_{t+k})]^k}, \tag{2}$$

where $E(\tilde{R}_{t+k})$ is the expected return on seats, the opportunity cost of the cash flow given its perceived riskiness. If $\tilde{\pi}_t$ follows a random walk it is easy to show that[4]

$$\frac{\tilde{PS}_t - PS_{t-1} + {}_{t-1}\pi^*_t}{PS_{t-1}} = E(\tilde{R}_t) + \frac{\tilde{a}_t}{PS_{t-1}} \sum_{k=1}^{\infty} \frac{1}{[1+E(\tilde{R}_{t+k})]^k}, \tag{3}$$

where $\tilde{a}_t$ is the unexpected change in profits from $t-1$ to t, which has a mean of zero. If $E(\tilde{R}_{t+k}) = E(\tilde{R}) > 0$ and ${}_t\pi^*_{t+k} = {}_t\pi^*$ for all $k > 0$, eq. (2) simplifies to

$$PS_t = \frac{{}_t\pi^*}{E(\tilde{R})}, \tag{4}$$

so that seat prices change over time only if expected future profits change. Thus, it is necessary to explain the behavior of profits in order to identify the determinants of changes in seat prices over time.

A simple model of the securities brokerage industry expresses the profits of the industry as a function of share trading volume, Q_t, and the level of the prices of the common stocks traded on the exchange, V_{St},

$$\pi_t = \{p_c(V_{St}) \cdot Q_t(p_c)\} - C(Q_t) \tag{5a}$$

$$= \pi(Q_t, V_{St}), \tag{5b}$$

where commission rates, p_c, are an increasing function of the level of stock prices[5] and brokers' costs, $C(Q_t)$, are a function of the level of share trading volume. This model obviously ignores many intricate aspects of the economics of the securities brokerage industry and the interrelated services which are

[4] The random walk assumption for profits is not restrictive; if profits are generated by a linear stochastic process [cf. Box and Jenkins (1970)], an expression similar to eq. (3) would relate unexpected changes in profits to changes in seat prices.

[5] The commission rate structure of the NYSE and ASE has always been an increasing function of stock prices and, until 1968, was not a function of the number of 100 share lots involved in a trade. Since 1968 various kinds of volume discounts have been instituted to make commission rates a decreasing function of the number of shares per trade. There have been very few changes in the commission rate structure over time [cf. Schwert (1975, pp. 20–25)].

provided by brokers, but it does illustrate some potentially important determinants of the demand for stock exchange seats. Based on (5), the expected future profits of brokers in time $t+k$, ${}_t\pi^*_{t+k}$, would be a function of the expected level of share volume, $E_t(\tilde{Q}_{t+k})$, and the expected level of share prices, $E_t(\tilde{V}_{St+k})$, where $E_t(\cdot)$ denotes the fact that the expectations are formed in period t.[6]

Assuming that the discount rate $E(R)$ is constant over time, the only thing that would cause seat prices to change over time would be changes in expectations of future share volume and stock prices. If the market for seats is efficient in its use of information only the *unexpected* changes in share volume and stock prices would cause unexpected changes in seat prices since the only new information about the future levels of share volume and stock prices which becomes available in period t is contained in these unexpected changes.[7] In order to test this model, conditional expectation predictors based on the past histories of share volume and stock prices, respectively, will be used to identify changes in these variables which are unexpected in each month from 1926–72.

4.1. *Time series models of share volume and stock prices*

The data which are used for these tests are: (1) the total NYSE and ASE monthly share trading volume as obtained from the respective exchanges,[8] and (2) a value-weighted index of NYSE stock prices.[9] The ACF's of the changes in the natural logarithms of these variables are listed in table 3, where r_{mt} is the continuously compounded return on the index of NYSE stocks, q_t and q_{at}

[6]In general, the expectation of future profits in (3) will also be a function of other moments. For example, if this is a constant average and marginal cost industry in each period and commission rates are a linear function of stock prices,

$$\begin{aligned} E_t(\tilde{\pi}_{t+k}) &= E_t\left\{\tilde{Q}_{t+k}\cdot\left[p_c(\tilde{V}_{st+k})-\frac{C(\tilde{Q}_{t+k})}{\tilde{Q}_{t+k}}\right]\right\} \\ &= E_t\{\tilde{Q}_{t+k}[\alpha_0+\alpha_1\tilde{V}_{st+k}-AC]\} \\ &= (\alpha_0-AC)E_t(\tilde{Q}_{t+k})+\alpha_1\,\mathrm{cov}_t(\tilde{Q}_{t+k},\tilde{V}_{st+k})+\alpha_1E_t(Q_{t+k})\cdot E_t(\tilde{V}_{st+k}) \\ &= [\alpha_0-AC+\alpha_1E_t(\tilde{V}_{st+k})]E_t(\tilde{Q}_{t+k})+\alpha_1\,\mathrm{cov}_t(\tilde{Q}_{t+k},\tilde{V}_{st+k}), \end{aligned}$$

so that the covariance between future share volume and future stock prices would also enter.

[7]Granger (1975) illustrates the mechanism by which expectations are updated in linear time series models when new information in the form of unexpected changes becomes available.

[8]The NYSE provided monthly data beginning in October 1934. For the period prior to October 1934, share volume data were obtained from *NYSE Yearbooks* and multiplied by 4/3 in order to get a definition of trading volume which is comparable to the post-1934 definition (an adjustment procedure which was suggested by the NYSE). The ASE provided monthly share volume data from 1927–72 inclusive, and no comparable data could be found for 1926.

[9]From 1926–46 the index is the Standard and Poor's composite index with dividends reinvested, and from 1947–72 the market portfolio is the value-weighted index of all NYSE common stocks compiled by the Center for Research in Security Prices at the University of Chicago. No monthly data on the level of ASE stock prices are readily available for the 1926–72 period, so the index of NYSE stock prices is used as a proxy.

Table 3

Autocorrelations of rates of change of share volume and stock prices.

	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	$\hat{\rho}_7$	$\hat{\rho}_8$	$\hat{\rho}_9$	$\hat{\rho}_{10}$	$\hat{\rho}_{11}$	$\hat{\rho}_{12}$	$S(\hat{\rho})$	$\bar{x}$	$\hat{S}$
1926–72															
r_{mt}	0.11	0.00	−0.15	0.05	0.08	−0.01	0.02	0.05	0.10	−0.02	−0.01	0.00	0.04	0.00793	0.06251
q_t	−0.13	−0.18	0.00	−0.16	−0.03	0.04	−0.05	0.05	0.05	−0.03	−0.04	0.12	0.04	0.00344	0.27222
q_{at}[a]	−0.14	−0.14	0.00	−0.13	0.00	−0.01	−0.06	0.03	0.07	−0.04	−0.03	0.12	0.04	0.00428	0.33241
1926–45															
r_{mt}	0.11	−0.01	−0.19	0.03	0.08	−0.01	0.04	0.09	0.11	0.02	−0.02	0.01	0.06	0.00614	0.08485
q_t	−0.07	−0.20	−0.04	−0.15	−0.08	0.07	−0.07	0.07	0.10	−0.04	−0.05	0.09	0.06	−0.00079	0.34775
q_{at}[a]	−0.08	−0.15	−0.02	−0.16	−0.04	0.02	−0.08	0.09	0.10	−0.02	−0.03	0.06	0.07	0.00326	0.39774
1946–72															
r_{mt}	0.09	0.02	−0.01	0.07	0.06	−0.03	−0.05	−0.09	0.07	−0.08	0.02	0.01	0.06	0.00926	0.03847
q_t	−0.27	−0.15	0.09	−0.17	0.11	−0.03	0.00	−0.02	−0.02	−0.01	−0.06	0.21	0.06	0.00658	0.19915
q_{at}	−0.23	−0.14	0.03	−0.08	0.07	−0.05	−0.03	−0.04	0.05	−0.08	−0.07	0.21	0.06	0.00500	0.27806

[a] ASE share volume data begins in 1927. $\bar{x}$ is the sample mean and $\hat{S}$ is the sample standard deviation.

are the rates of change of NYSE and ASE share volume, respectively. As with the rates of change of seat prices, the variability of these series is much greater in the 1926–45 period than in the 1946–72 period as indicated by the sample standard deviations.

Although the ACF for r_{mt} is close to zero at all lags through 12 in the subperiods, there appears to be significant autocorrelation in q_t and q_{at} at lags 1, 2, and 4 in both periods. This suggests that the marginal distribution of $\tilde{r}_{mt}$ is essentially the same as the conditional distribution based upon the past history of the series; however, the marginal and conditional (on the past history of the series) distributions of $\tilde{q}_t$ and $\tilde{q}_{at}$ are not the same. In particular, the conditional expectation of $\tilde{r}_{mt}$ at time t is the same as the marginal expectation, but the conditional expectations of $\tilde{q}_t$ and $\tilde{q}_{at}$ depend on past realizations of q_t and q_{at}, respectively.

The ACF's for q_t and q_{at} suggest that they may be fourth-order moving average processes and this impression is borne out by the estimated autoregressive-integrated-moving average [ARIMA(p, d, q)][10] models for q_t and q_{at} in table 4. It seems that both q_t and q_{at} can be adequately represented as an MA(4) process with the third moving average parameter, θ_3, not significantly different from zero. The Box–Pierce statistic for 24 lags of the residual ACF, $Q^{\hat{u}}(24)$, which is asymptotically distributed as χ^2_{21} in this case, indicates that we cannot reject the hypothesis that the MA(4) model is an adequate model.[11] It should be noted that the residual variance for both series is more than twice as large in the 1926–45 period as in the 1946–72 period, although the MA parameters are similar in the two subperiods.

In the usual treatment of ARIMA processes it is assumed that the noise process, which is the unexpected part of each variable at time t, has a stationary normal distribution which is independent through time so that all of the information about the noise distribution is summarized in its sufficient statistics, the mean and variance. The S.R. statistics for the residuals from these ARIMA models are consistent with the hypothesis of normality except for the model for ASE share volume in the latter period. Thus, the extrapolative time series models seem to provide a functional way to separate the monthly percent change in share trading volume into 'expected' and 'unexpected' parts: the residials from these models seem to behave like serially incorrelated normal variables and would be unexpected based on the past history of the series. The similarity of the MA models for the rates of change in NYSE and ASE share volume and the constancy of the MA parameters over time is an indication that these models might provide useful proxies for the market's expectations of share volume.

[10]Box and Jenkins (1970) describe the ARIMA (p, d, q) model in detail where p is the order of the AR polynomial in the lag operator L: $L^k Z_t = Z_{t-k}$, d is the order of differencing required to induce mean stationarity, and q is the order of the MA polynomial.

[11]Other MA models were estimated including θ_3 or θ_5, but these alternatives did not significantly improve the explanatory power of the model.

Table 4

Moving average models for rates of change in NYSE and ASE share volume (asymptotic standard errors in parentheses).

Period	Constant $\hat{c}$	Moving average parameters $\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_4$	$S(\hat{a})$	R^2	$Q^{\hat{a}}(24)$	S.R.$(\hat{a})$
(A) *NYSE share volume*: $q_t = \hat{c}+[1-\hat{\theta}_1L-\hat{\theta}_2L^2-\hat{\theta}_4L^4]\hat{a}_t$								
1926–45	−0.0011 (0.0071)	0.2333 (0.0602)	0.2840 (0.0660)	0.1538 (0.0626)	0.3289	0.116	22.1	6.09
1946–72	0.0068 (0.0031)	0.3358 (0.0543)	0.2025 (0.0553)	0.1596 (0.0515)	0.1821	0.171	36.8	5.82
(B) *ASE share volume*: $q_{at} = \hat{c}+[1-\hat{\theta}_1L-\hat{\theta}_2L^2-\hat{\theta}_4L^4]\hat{a}_{at}$								
1927–45	0.0024 (0.0124)	0.1644 (0.0640)	0.2063 (0.0671)	0.1485 (0.0663)	0.3846	0.078	16.6	6.13
1946–72	0.0055 (0.0055)	0.3159 (0.0542)	0.1960 (0.0562)	0.1158 (0.0521)	0.2613	0.124	30.1	8.65[a]

[a]S.R. exceeds the 0.95 fractile of the sampling distribution when sampling from a normal population.

4.2. Relationships of seat prices with stock prices

The cross correlation functions (CCF's) between r_t or r_{at} and r_{mt} in table 5 suggest that the contemporaneous relationship between unexpected changes in seat prices and unexpected changes in stock prices is quite strong.[12] There also seems to be a relationship between current seat price changes and the previous month's change in stock prices, especially for ASE data, but this correlation may be due to the fact that seats are not always traded on the last day of every month. This question will be analyzed in detail within the context of the time series regression models below.

The CCF analysis suggests that the 'market model' regression of $\tilde{r}_t$ or $\tilde{r}_{at}$ on $\tilde{r}_{mt}$ would yield a significant statistical relationship. Table 6 contains estimates of this regression for the 1926–72 period and the 1926–45 and 1946–72 subperiods and tests for the stability of the regression coefficients over time based on the F-statistic[13]

$$F_{k_0,n-k-k_0} = \frac{(Q_1 - Q_2)/k_0}{Q_2/[n-k\cdot m]}, \tag{6}$$

Q_1 = sum-of-squares (ss) from the regression for the entire period,
$Q_2 = \sum_{i=1}^{m} ss_i$: sum of the sums-of-squares from subperiod regressions,
m = number of mutually exclusive and exhaustive subperiods,
n = sample size,
k = number of regression coefficients to be estimated including the intercept,
$k_0 = (m-1)\cdot k$.

These estimated regression equations indicate that there is a relationship between $\tilde{r}_t$ or $\tilde{r}_{at}$ and $\tilde{r}_{mt}$, and the hypothesis that the regression parameters are constant over the 1926–72 period cannot be rejected at usual significance levels. As before, the residual variance for this model is larger in the 1926–45 period than in the 1946–72 period.[14]

It should be noted that the slope coefficient, β, in this market model regression equation can be used to infer the riskiness of stock exchange seats in the context of the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), Black (1970), and others. In the CAPM the expected return on an asset is a linear function of the relative risk of the asset in the value-weighted portfolio of all marketable assets, for example,

$$E(\tilde{R}_{it}) = R_{ft} + [E(\tilde{R}_{mt}) - R_{ft}]\beta_i, \tag{7}$$

[12]Box and Jenkins (1970) recommend the CCF as a means of identifying a tentative specification of time series regression models.

[13]This statistic has an F-distribution if the regression disturbances have a normal distribution with homoscedastic variance. Since there is evidence of heteroscedasticity during the 1926–72 period, these statistics should be interpreted with caution.

[14]These regressions were estimated for nonoverlapping eight year subperiods within the 1926–72 period and there was no evidence of instability of the regression coefficients.

Table 5

Cross correlations of unexpected changes in seat prices with unexpected changes in stock prices and share volume: $\hat{\rho}(r_t, X_{t+k})$.

X_{t+k}	k: −6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	$S(\hat{\rho})$[a]
(A) *NYSE seat prices*, r_t														
1926–45														
r_{mt+k}	0.04	−0.08	−0.16	−0.06	0.03	0.20	0.52	0.13	0.04	−0.03	−0.05	0.03	−0.06	0.06
u_{t+k}	−0.06	0.00	−0.03	−0.16	0.03	0.11	0.36	0.18	0.10	0.01	−0.02	−0.01	0.05	0.06
1946–72														
r_{mt+k}	−0.02	0.17	0.03	−0.08	−0.04	0.25	0.43	0.08	0.04	0.00	0.04	−0.03	0.03	0.06
u_{t+k}	−0.01	0.02	−0.01	0.10	−0.08	0.15	0.25	0.17	−0.03	0.03	0.05	0.01	0.02	0.06
(B) *ASE seat prices*, r_{at}														
1927–45														
r_{mt+k}	0.09	0.06	−0.13	−0.03	0.02	0.37	0.36	0.07	0.00	−0.04	−0.01	−0.01	−0.03	0.07
u_{at+k}	0.02	−0.02	−0.04	0.13	0.07	0.19	0.38	0.16	0.03	0.07	0.00	0.03	−0.02	0.07
1946–72														
r_{mt+k}	−0.01	0.06	0.07	−0.03	0.19	0.32	0.23	0.04	−0.03	0.05	−0.04	0.01	−0.13	0.06
u_{at+k}	−0.08	0.10	0.05	0.00	0.17	0.19	0.19	0.08	0.04	0.09	−0.03	0.02	−0.03	0.06

[a] $S(\hat{\rho}) = n^{-\frac{1}{2}}$ is the asymptotic standard error of the cross correlation coefficient between two 'white noise' series under the null hypothesis of no correlation.

Table 6

'Market model' regressions: $r_t = \hat{a} + \hat{\beta} r_{mt} + \hat{\varepsilon}_t$ (standard errors in parentheses).

Period	$\hat{a}$	$\hat{\beta}$	$S(\hat{\varepsilon})$	R^2	$Q^{e}(24)$	$Q^{rm}(24)$	S.R.$(\hat{\varepsilon})$	
			(A) *NYSE seat prices*, r_t					
1926–72	−0.0067 (0.0046)	0.9532 (0.0728)	0.1080	0.234	52.0	43.2	8.95[a]	
1926–45	−0.0062 (0.0082)	0.9018 (0.0964)	0.1265	0.269	34.2	31.3	7.70[a]	$F_{2,560} = 0.90$
1946–72	−0.0086 (0.0053)	1.138 (0.1331)	0.0920	0.185	40.1	48.1	6.71[b]	
			(B) *ASE seat prices*, r_{at}					
1926–72	−0.0058 (0.0073)	0.9441 (0.1167)	0.1730	0.104	36.9	92.7	10.0[a]	
1926–45	−0.0060 (0.0140)	0.9844 (0.1648)	0.2162	0.130	21.8	48.0	8.02[a]	$F_{2,560} = 0.21$
1946–72	−0.0044 (0.0076)	0.7989 (0.1923)	0.1330	0.051	52.0	58.9	8.49[a]	

[a]S.R. exceeds the 0.99 fractile of the sampling distribution when sampling from a normal population.
[b]S.R. exceeds the 0.95 fractile of the sampling distribution when sampling from a normal population.

where

$$R_{it} = \left(\frac{\tilde{P}_{it} - P_{it-1} + \tilde{D}_{it}}{P_{it-1}}\right)$$

is the one-period rate of return on asset i, $\tilde{D}_{it}$ is the cash flow in period t to the holder of asset i, R_{ft} is the rate of return on a risk-free asset in period t, $\tilde{R}_{mt}$ is the rate of return on the market portfolio, and the measure of relative risk is defined as:

$$\beta_i = \operatorname{cov}(\tilde{R}_{it}, \tilde{R}_{mt})/\sigma^2(\tilde{R}_{mt}).$$

Since the rates of change in seat prices, r_t, and stock prices, r_{mt}, closely approximate the discrete percentage changes in these prices, the relative risk of stock exchange seats, β_i, is related to the slope coefficients estimated in table 6, β,

$$\beta_i = \operatorname{cov}\left(\frac{\tilde{PS}_t - PS_{t-1}}{PS_{t-1}} + \frac{\tilde{\pi}_t}{PS_{t-1}}, \tilde{R}_{mt}\right)\Big/\sigma^2(\tilde{R}_{mt}), \tag{8a}$$

$$\approx \beta + \operatorname{cov}\left(\frac{\tilde{\pi}_t}{PS_{t-1}}, \tilde{R}_{mt}\right)\Big/\sigma^2(\tilde{R}_{mt}), \tag{8b}$$

where $\tilde{\pi}_t$ is the cash flow received by a seat-holder. If the cash flows received by seat-holders follow a random walk, so that $E_t(\tilde{\pi}_{t+k}) = \pi_t$ for all $k > 0$, eq. (4) shows that $\tilde{\pi}_t = E(\tilde{R}_i)\cdot\tilde{PS}_t$ so that

$$\beta_i \approx \beta\cdot[1+E(\tilde{R}_i)] > \beta, \tag{9a}$$

and

$$E(\tilde{R}_{it}) \approx \frac{R_{ft} + \beta[E(\tilde{R}_{mt}) - R_{ft}]}{\{1 - \beta[E(\tilde{R}_{mt}) - R_{ft}]\}} > R_{ft} + \beta[E(\tilde{R}_{mt}) - R_{ft}], \tag{9b}$$

for $\beta > 0$. Thus, for monthly data plausible values of $E(\tilde{R}_i)$ such as 0.01 would indicate that the bias in using $\hat{\beta}$ as an estimate of β_i may be quite small, so that the expected return on the seat is approximately

$$E(\tilde{R}_i) = R_f + [E(\tilde{R}_m) - R_f]\beta. \tag{10}$$

This analysis indicates that seats are about as risky as the market index itself (since $\hat{\beta} \approx 1$) so that the expected return on seats would be close to the expected return on the value-weighted index of NYSE common stocks.

Box and Jenkins (1970) suggest the use of the residual ACF and the CCF between the regression residuals and each exogenous variable as a means of identifying omitted lagged values of the endogenous variable, or the exogenous variable, or the residuals in a dynamic regression model. Box–Pierce statistics can be used as summary measures of the smallness of these functions where $Q^e(24)$ represents the statistic fot 24 lags of the residual ACF and $Q^X(24)$ represents the statistic for 24 lags of the CCF between the residuals and exogenous variable X. These diagnostic checks, which are asymptotically distributed as χ^2_{23} variables in this case, are large for the subperiods 1926–45 and 1946–72 indicating that lagged values of r_{mt} and $\hat{e}_t$ have been incorrectly omitted in the market model regressions.

In order to test whether the relationship between r_t or r_{at} and r_{mt} is strictly contemporaneous, r_t and r_{at} are regressed on r_{mt} and r_{mt-1} with an MA(1) noise process using nonlinear least squares.[15] The results of these regressions are reported in table 7. These estimates indicate that both β_1, the coefficient of r_{mt-1}, and θ_1, the moving average parameter for the disturbance, are statistically significant by conventional standards. In fact, $\hat{\beta}_1$ is considerably larger than $\hat{\beta}_0$ for ASE data in the 1946–72 period.

It is possible that the definition of r_t and r_{at} using the last transaction price in each month may cause a spurious relationship with r_{mt-1} if the date of the last seat transaction in a month is not near the end of the month. Since the returns on the market index are measured on the last day of each month, there could be some instances where the last seat trade would occur closer to r_{mt-1} than r_{mt} in time. For example, if the last seat trade in a month occurs on the third day of the month and the last seat trade in the next month occurs on the last day of the month the price change between these months would be related to the return on the market for *both* months.[16] This question is examined in detail for NYSE seat prices over the 1952–72 period in section 5 since the lagged response of seat prices to unexpected changes in stock prices would be inconsistent with market efficiency unless the phenomenon is spurious.

As Ibbotson (1975, p. 242) has noted, the sum of the coefficients of the contemporaneous and lagged returns to the market index may provide a better estimate of the relative risk of an asset when trading is infrequent. Therefore, $(\hat{\beta}_0+\hat{\beta}_1)$ from table 7 may provide a more accurate picture of the relative risk of NYSE and ASE seats as assets in diversified portfolios.[17] Based on the estimates in table 7 it seems that ASE seats are substantially more risky than NYSE seats because the values of $\hat{\beta}_1$ for ASE data are relatively large. This is consistent

[15]The known statistical properties of these regression equations are based upon asymptotic results; cf., Pierce (1972). Note that the statistic for regression parameter stability does not have an exact F-distribution in this case even when the disturbances are homoscedastic.

[16]Fama (1965b) analyzed the Fisher effect on the autocorrelations of returns to portfolios of common stocks.

[17]Joseph Williams has pointed out that it is possible to get a better estimate of the relative risk of an asset by taking account of the autocorrelation in r_{mt} and by considering the relationship between current returns on the individual asset and future returns on the market index which are induced by the non-trading of some of the assets in the market index. These considerations are not likely to be important in this case because all of the assets in the NYSE stock index are traded much more frequently than stock exchange seats.

Table 7

Extended 'market model' regressions: $r_t = \hat{a} + [\hat{\beta}_0 + \hat{\beta}_1 L] r_{mt} + [1 - \hat{\theta}_1 L] \hat{\varepsilon}_t$ (asymptotic standard errors in parentheses).

Period	$\hat{a}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\theta}_1$	$S(\hat{\varepsilon})$	R^2	$Q^a(24)$	$Q^{rm}(24)$	S.R.($\hat{\varepsilon}$)	
(A) *NYSE seat prices*, r_t										
1926–72	−0.0093 (0.0032)	0.9660 (0.0708)	0.3078 (0.0703)	0.2787 (0.0409)	0.1039	0.295	46.1	28.0	8.78[a]	
1926–45	−0.0080 (0.0063)	0.9100 (0.0956)	0.2528 (0.0947)	0.2186 (0.0642)	0.1237	0.310	33.8	26.1	7.60[a]	$F_{4,555} = 1.5$
1946–72	−0.0129 (0.0033)	1.145 (0.1251)	0.4944 (0.1245)	0.3540 (0.0525)	0.0858	0.297	24.3	32.2	6.47	
(B) *ASE seat prices*, r_{at}										
1926–72	−0.0125 (0.0057)	0.8655 (0.1096)	0.9136 (0.1095)	0.1767 (0.0417)	0.1616	0.223	37.0	27.7	10.4[a]	
1926–45	−0.0110 (0.0114)	0.9066 (0.1558)	0.8497 (0.1556)	0.1407 (0.0647)	0.2029	0.243	25.5	20.3	8.36[a]	$F_{4,555} = 0.70$
1946–72	−0.0146 (0.0054)	0.7008 (0.1794)	1.193 (0.1787)	0.2569 (0.0542)	0.1231	0.194	41.5	23.2	8.53[a]	

[a]S.R. exceeds the 0.99 fractile of the sampling distribution when sampling from a normal population.

with the casual observation that the estimated dispersion in the unexpected rates of change in ASE share volume is greater than for NYSE data in table 4 and the standard deviation of r_{at} is greater than the standard deviation of r_t in table 2. Using the sum of $\hat{\beta}_0$ and $\hat{\beta}_1$ as an estimate of relative risk, the evidence in table 7 suggests that stock exchange seats are probably more risky than the value-weighted index of NYSE-listed common stocks; that is $(\hat{\beta}_0+\hat{\beta}_1)$ is significantly greater than unity in the 1926–45 and 1946–72 periods.

4.3. Relationships of seat prices with share volume

As noted above, the only information about expectations of future share volume which is new at time t is contained in the current *unexpected* change in share volume. If the market is efficient in processing information about the value of seats, only the contemporaneous unexpected percent change in share volume, $\hat{u}_t$ or $\hat{u}_{at}$, should affect r_t or r_{at}, respectively. However, the CCF's between rates of change in seat prices and unexpected rates of change of share volume in table 5 suggest that there may be a relationship between current changes in seat prices and lagged unexpected changes in share volume, especially for ASE data.

One test of the efficient market hypothesis is whether current *expected* percent changes in share volume, $\hat{q}_t$, are associated with changes in seat prices since this information would be known at time $t-1$. The estimates of the regression models for this test are listed in table 8, and for NYSE data the estimates of the coefficient of the current *unexpected* change in share volume, $\hat{\gamma}_1$, are statistically significant over long periods and relatively constant across shorter time periods while the estimates of the coefficient of the *expected* change in share volume, $\hat{\gamma}_2$, have relatively large standard errors and change erratically across the shorter subperiods. Thus, the market for NYSE seats seems to react to the new information contained in the current *unexpected* change in share volume while ignoring the old information in the form of the expected change in NYSE share volume.

The results for ASE data are more difficult to interpret. In the 1927–45 period and the subperiods within that time, $\hat{\gamma}_1$ is statistically significant based on its asymptotic normal distribution while $\hat{\gamma}_2$ is not significant, so the ASE results corroborate the NYSE conclusions in the earlier period. However, in the 1946–72 period, and especially the 1966–72 subperiod, it seems as though expected increases in ASE share volume had a significant *negative* impact on the value of ASE seats. This result is counter-intuitive since it is inconsistent with market efficiency and it implies that ASE seat-holders could increase the value of their future cash flows by reducing current share volume.

Fortunately, there is another plausible explanation for the significant negative value of $\hat{\gamma}_2$. The current forecast of the rate of change of ASE share volume is derived from the equation

$$\hat{q}_{at} = \hat{c}-\hat{\theta}_1\hat{u}_{at-1}-\hat{\theta}_2\hat{u}_{at-2}-\hat{\theta}_4\hat{u}_{at-4}, \tag{11}$$

Table 8

Share volume effects: $r_t = \hat{a} + [\hat{\beta}_0 + \hat{\beta}_1 L]r_{mt} + \hat{\gamma}_1 \hat{u}_t + \hat{\gamma}_2 \hat{q}_t + [1 - \hat{\theta}_1 L]\hat{\varepsilon}_t$ (asymptotic standard errors in parentheses).

Period	$\hat{a}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\theta}_1$	$S(\hat{\varepsilon})$	R^2	$Q^{\hat{\varepsilon}}(24)$	$Q^{r_m}(24)$	$Q^{\hat{u}}(24)$	$Q^{\hat{q}}(24)$	S.R.($\hat{\varepsilon}$)	
(A) *NYSE data*														
1926–72	−0.0075 (0.0030)	0.8838 (0.0703)	0.1638 (0.0740)	0.1065 (0.0179)	−0.0027 (0.0358)	0.3213 (0.0404)	0.1009	0.337	37.4	30.9	23.0	27.3	8.32[a]	
1926–45	−0.0066 (0.0058)	0.8318 (0.0939)	0.1001 (0.0993)	0.1135 (0.0255)	−0.0020 (0.0525)	0.2567 (0.0638)	0.1192	0.364	32.2	29.5	20.3	19.7	7.19[a]	$F_{6,551} = 1.4$
1946–72	−0.0106 (0.0032)	1.040 (0.1269)	0.3762 (0.1317)	0.0944 (0.0274)	−0.0073 (0.0534)	0.3961 (0.0516)	0.0845	0.323	24.0	32.4	23.1	21.4	6.33	
1926–33	−0.0020 (0.0111)	0.7603 (0.1155)	0.0818 (0.1261)	0.1273 (0.0425)	−0.1569 (0.0952)	0.0423 (0.1074)	0.1131	0.498	26.8	49.4	41.3	46.1	4.77	
1934–41	−0.0198 (0.0084)	0.8307 (0.1834)	0.0575 (0.1883)	0.1076 (0.0378)	0.0371 (0.0796)	0.3899 (0.0995)	0.1290	0.334	15.1	9.5	15.0	15.2	6.59[a]	
1942–49	−0.0083 (0.0077)	1.111 (0.2176)	0.2318 (0.2289)	0.0325 (0.0377)	0.0387 (0.0764)	0.2700 (0.1024)	0.0911	0.343	11.7	13.3	23.1	21.6	4.92	
1950–57	−0.0013 (0.0066)	0.4828 (0.2113)	0.0421 (0.2308)	0.1699 (0.0425)	−0.0870 (0.0771)	0.2324 (0.1039)	0.0684	0.266	27.6	20.4	17.1	14.6	5.73	$F_{30,527} = 1.6$
1958–65	−0.0020 (0.0037)	1.206 (0.2517)	0.0563 (0.2520)	0.0856 (0.0473)	−0.0527 (0.0936)	0.6378 (0.0837)	0.0743	0.414	14.2	21.4	12.1	15.2	5.45	
1966–72	−0.0175 (0.0069)	1.243 (0.2929)	0.8340 (0.3120)	0.0754 (0.0918)	0.0769 (0.1736)	0.4374 (0.1040)	0.1073	0.403	17.1	35.6	24.1	21.8	4.93	
(B) *ASE data*														
1927–72	−0.0080 (0.0054)	0.7185 (0.1077)	0.6719 (0.1135)	0.1089 (0.0227)	−0.1947 (0.0573)	0.2004 (0.0420)	0.1546	0.272	25.0	30.9	27.6	19.7	10.7[a]	
1927–45	−0.0069 (0.0108)	0.7354 (0.1512)	0.5909 (0.1603)	0.1593 (0.0365)	−0.1400 (0.1052)	0.1642 (0.0664)	0.1925	0.311	18.7	23.1	21.5	14.1	8.55[a]	$F_{6,539} = 2.0$
1946–72	−0.0095 (0.0051)	0.6558 (0.1819)	0.8684 (0.1911)	0.0299 (0.0281)	−0.2576 (0.0600)	0.3045 (0.0536)	0.1196	0.243	43.0	17.4	27.8	27.4	8.22[a]	
1927–33	0.0005 (0.0119)	0.8219 (0.1577)	0.4258 (0.1749)	0.1437 (0.0498)	−0.0553 (0.1271)	0.3237 (0.1097)	0.1582	0.516	13.2	23.7	17.8	15.8	4.59	
1934–41	−0.0379 (0.0179)	0.4400 (0.2784)	0.7292 (0.2856)	0.1702 (0.0580)	−0.0315 (0.1809)	0.1319 (0.1054)	0.1970	0.257	10.4	17.6	19.0	18.3	6.72[a]	
1942–49	−0.0021 (0.0182)	1.164 (0.4927)	0.9715 (0.5201)	0.1545 (0.0727)	−0.2035 (0.1809)	0.2420 (0.1034)	0.2000	0.274	17.1	11.8	17.6	13.3	6.19	
1950–57	−0.0062 (0.0087)	0.3932 (0.2950)	0.7051 (0.3241)	0.0200 (0.0456)	−0.1642 (0.1001)	0.2531 (0.1035)	0.0945	0.200	15.7	13.9	16.8	17.8	8.30[a]	$F_{30,515} = 1.5$
1958–65	−0.0020 (0.0083)	0.4732 (0.2964)	0.7354 (0.2993)	0.0185 (0.0441)	−0.0220 (0.0926)	0.1771 (0.1060)	0.0858	0.147	20.4	21.9	16.0	15.6	5.95	
1966–72	−0.0097 (0.0106)	0.7931 (0.3836)	1.181 (0.3964)	−0.0163 (0.0579)	−0.4897 (0.1236)	0.3380 (0.1091)	0.1396	0.378	23.4	19.4	12.4	16.9	5.44	

[a] S.R. exceeds the 0.99 fractile of the sampling distribution when sampling from a normal population.

based on the MA(4) model for q_{at} in table 4. Thus, $\hat{q}_{at}$ is just a linear combination of the four most recent values of $\hat{u}_{at}$ with negative weights. The CCF between r_{at} and $\hat{u}_{at}$ in table 5 indicates a relatively strong positive relationship between r_{at} and both $\hat{u}_{at-1}$ and $\hat{u}_{at-2}$ during the 1946–72 period. This lagged relationship may be due to the infrequent trading of ASE seats during this period. Because both $\hat{u}_{at-1}$ and $\hat{u}_{at-2}$ are omitted from the regression equations in table 8, the positive relationship between r_{at} and both $\hat{u}_{at-1}$ and $\hat{u}_{at-2}$ may be reflected in the negative estimate of the coefficient of $\hat{q}_{at}$, since $\hat{q}_{at}$ is negatively related to $\hat{u}_{at-1}$ and $\hat{u}_{at-2}$ as indicated in (11). Thus, the seemingly significant negative coefficient for $\hat{q}_{at}$ in the 1946–72 period may simply reflect a significant positive relationship between current values of r_{at} and lagged values of $\hat{u}_{at}$ due to the Fisher effect during this period.

In order to provide direct evidence about the relative importance of the Fisher effect for NYSE and ASE data in different time periods, current rates of change in seat prices are regressed on current and lagged values of the unexpected rate of change in share volume and stock prices with an MA(1) noise process,

$$r_t = \hat{\alpha} + [\hat{\beta}_0 + \hat{\beta}_1 L] r_{mt} + [\hat{\gamma}_0 + \hat{\gamma}_1 L] \hat{u}_t + [1 - \hat{\theta}_1 L] \hat{e}_t. \tag{12}$$

The results for this test of the non-trading phenomenon are listed in table 9. As predicted by the CCF's in table 5, the effect of lagged unexpected changes in share volume as estimated by $\hat{\gamma}_1$ is greatest for ASE data in the 1946–72 period, and especially the 1966–72 period. The magnitudes of $\hat{\beta}_1$ and $\hat{\gamma}_1$, the estimates of the coefficients of lagged unexpected changes in stock prices and share volume, relative to $\hat{\beta}_0$ and $\hat{\gamma}_0$ suggest that the non-trading phenomenon is most serious in the ASE seat price data during these periods. Unfortunately, ASE seat price data could not be obtained in more detail than the last-trade-in-the-month series, so the direct tests of the Fisher effect in section 5 must be restricted to NYSE data.

This analysis of the relationship between rates of change in stock exchange seat prices and unexpected changes in stock prices and share trading volume indicates that the simple model of securities brokers' profitability in (5) may be a useful approximation to reality. The regression models in table 9 explain about a third of the time series variation of r_t and r_{at}, and the diagnostic checks do not indicate obvious model misspecifications from a statistical viewpoint. The next section of the paper provides evidence on the extent to which the significance of the coefficients of the lagged values of r_{mt} is due to the fact that seats are not actively traded. This evidence answers some of the remaining questions about the efficiency of the market for stock exchange seats.

5. Test of the effect of inactive seat trading on the NYSE market model

In order to get a direct test of the 'Fisher effect' on the relationship between $\tilde{r}_t$ and $\tilde{r}_{mt}$, the Standard and Poor's composite stock price index is measured on

Table 9

Fisher effects for share volume and stock prices: $r_t = \hat{a} + [\hat{\beta}_0 + \hat{\beta}_1 L] r_{mt} + [\hat{\gamma}_0 + \hat{\gamma}_1 L]\hat{a}_t + [1 - \hat{\theta}_1 L]\hat{\varepsilon}_t$ (asymptotic standard errors in parentheses).

Period	$\hat{a}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\theta}_1$	$S(\hat{\varepsilon})$	R^2	$Q^{\varepsilon}(24)$	$Q^{rm}(24)$	$Q^{\hat{a}}(24)$	S.R.$(\hat{\varepsilon})$	
						(A) *NYSE data*							
1926–72	−0.0073 (0.0030)	0.8827 (0.0702)	0.1399 (0.0739)	0.1004 (0.0182)	0.0263 (0.0173)	0.3165 (0.0404)	0.1007	0.340	37.6	32.5	20.3	8.44[a]	
1926–45	−0.0065 (0.0058)	0.8300 (0.938)	0.0825 (0.0995)	0.1092 (0.0258)	0.0214 (0.0242)	0.2522 (0.0639)	0.1190	0.366	31.7	29.6	19.3	7.30[a]	$F_{6,551} = 1.5$ (1926–45 and 1946–72)
1946–72	−0.0103 (0.0032)	1.040 (0.1265)	0.3376 (0.1301)	0.0854 (0.0277)	0.0357 (0.0269)	0.3924 (0.0517)	0.0843	0.327	25.6	34.8	22.8	6.34	
1926–33	−0.0017 (0.0097)	0.7656 (0.1144)	0.0287 (0.1294)	0.1366 (0.0428)	0.0836 (0.0387)	0.1600 (0.1060)	0.1120	0.508	25.2	47.3	38.4	4.77	$F_{30,527} = 1.6$ (1926–33 through 1966–72)
1934–41	−0.0189 (0.0085)	0.8251 (0.1834)	−0.0634 (0.1879)	0.1016 (0.0400)	0.0052 (0.0397)	0.3793 (0.0999)	0.1291	0.333	14.7	10.3	15.0	6.73[a]	
1942–49	−0.0079 (0.0077)	1.111 (0.2180)	0.2111 (0.2237)	0.0333 (0.0386)	0.0116 (0.0365)	0.2687 (0.1024)	0.0912	0.342	11.7	13.9	23.5	4.83	
1950–57	−0.0010 (0.0066)	0.4698 (0.2081)	0.0000 (0.2294)	0.1630 (0.0422)	0.0817 (0.0389)	0.2179 (0.1042)	0.0673	0.290	25.8	18.3	15.0	5.82	
1958–65	−0.0023 (0.0036)	1.206 (0.2512)	−0.0606 (0.2451)	0.0765 (0.0496)	0.0411 (0.0501)	0.6342 (0.0840)	0.0741	0.416	14.1	21.5	12.1	5.47	
1966–72	−0.0173 (0.0068)	1.249 (0.2922)	0.8244 (0.3148)	0.0753 (0.0919)	0.0318 (0.0875)	0.4437 (0.1036)	0.1073	0.403	17.1	35.3	24.2	4.91	
						(B) *ASE data*							
1927–72	−0.0087 (0.0053)	0.7206 (0.1075)	0.6460 (0.1143)	0.1047 (0.0227)	0.0797 (0.0215)	0.2128 (0.0419)	0.1543	0.274	25.7	28.7	27.2	10.7[a]	
1927–45	−0.0072 (0.0105)	0.7353 (0.1499)	0.5422 (0.1608)	0.1537 (0.0364)	0.0787 (0.0341)	0.1757 (0.0663)	0.1910	0.322	17.7	24.0	17.7	8.57[a]	$F_{6,539} = 1.6$ (1927–45 and 1946–72)
1946–72	−0.0116 (0.0052)	0.6411 (0.1845)	0.9442 (0.1938)	0.0329 (0.0286)	0.0809 (0.0274)	0.2983 (0.0538)	0.1214	0.221	44.0	18.9	35.6	8.16[a]	
1927–33	0.0008 (0.0116)	0.8247 (0.1561)	0.3705 (0.1773)	0.1385 (0.0495)	0.0578 (0.0444)	0.3302 (0.1092)	0.1567	0.525	13.5	21.3	15.7	5.18	$F_{30,515} = 1.3$ (1927–33 through 1966–72)
1934–41	−0.0375 (0.0176)	0.4413 (0.2776)	0.7130 (0.2850)	0.1635 (0.0585)	0.0423 (0.0579)	0.1356 (0.1053)	0.1964	0.261	10.4	19.7	17.8	6.76[a]	
1942–49	−0.0045 (0.0175)	1.149 (0.4948)	1.030 (0.5018)	0.1500 (0.0732)	0.0737 (0.0664)	0.2527 (0.1031)	0.2000	0.273	17.0	12.5	18.7	6.12	
1950–57	−0.0063 (0.0085)	0.4118 (0.2929)	0.6720 (0.3221)	0.0163 (0.0453)	0.0849 (0.0426)	0.2666 (0.1031)	0.0939	0.211	13.7	13.0	16.0	8.20[a]	
1958–65	−0.0021 (0.0084)	0.4621 (0.2975)	0.7579 (0.3013)	0.0213 (0.0451)	0.0026 (0.0420)	0.1735 (0.1060)	0.0858	0.147	20.3	21.9	15.8	5.98	
1966–72	−0.0110 (0.0109)	0.6960 (0.3974)	1.208 (0.4243)	0.0048 (0.0599)	0.1672 (0.0579)	0.3484 (0.1089)	0.1457	0.324	21.3	16.9	15.2	5.33	

[a] S.R. exceeds the 0.99 fractile of the sampling distribution when sampling from a normal population.

each of the dates when NYSE seat trades occurred during the period from June 1952 through December 1972.[18] The continuously compound return on the Standard and Poor's index is computed over the same intervals as the actual time which elapsed between the last seat trade in successive months from 1952–72. Thus, the matched-dates series of returns on the Standard and Poor's index, r'_{mt}, is measured synchronously with the monthly rate of change in NYSE seat prices, r_t. The Standard and Poor's index is also measured on the last day of each month in order to compute the actual monthly continuously compounded return on the index, r''_{mt}.

If the market for seats is efficient in assimilating the information contained in changes in stock prices into the prices of stock exchange seats, the coefficient of r'_{mt-1} in eq. (13a) should not be significantly different from zero; however, if the Fisher effect (due to the infrequent trading of seats) is a serious problem the coefficient of r''_{mt-1} might be significant in the regression eq. (13b),

$$r_t = \alpha + [\beta_0 + \beta_1 L] r'_{mt} + \varepsilon_t, \tag{13a}$$

$$r_t = \alpha + [\gamma_0 + \gamma_1 L] r''_{mt} + \varepsilon_t. \tag{13b}$$

Thus, if $\hat{\gamma}_1$ is significantly different from zero and $\hat{\beta}_1$ is not, we would have direct evidence that the significance of the lagged values of r_{mt}, $\hat{u}_t$, and $\hat{\varepsilon}_t$ in tables 7–9 is attributable to the non-synchronous measurement of seat prices, stock prices, and share volume and is not evidence of an inefficient market for stock exchange seats.

The estimates of eq. (13) for the August 1952 – December 1972 period, and for the August 1952 – December 1962 and January 1963 – December 1972 subperiods, in table 10 suggest that the Fisher effect is the cause of the significant lagged values of r_{mt} and $\hat{u}_t$ which had been observed in section 4. The regressions of r_t on the matched dates series of market returns r'_{mt}, always have a smaller standard error, $S(\hat{\varepsilon})$, and a smaller Box–Pierce statistic for the residual ACF, $Q^*(12)$, than the regressions of r_t on the end-of-month series of market returns, r''_{mt}. In addition, the estimates of β_0 are always greater than the estimates of γ_0, while the values of $\hat{\beta}_1$ are smaller than the values of $\hat{\gamma}_1$ in both absolute magnitude and statistical significance. $\hat{\beta}_1$ is never significantly different from zero based on a test at the 5 percent level, but $\hat{\gamma}_1$ is highly significant in the 1952–72 and 1963–72 periods.

Interestingly, however, the sum $(\hat{\beta}_0 + \hat{\beta}_1)$ is always quite close to $(\hat{\gamma}_0 + \hat{\gamma}_1)$, suggesting that the use of lagged values of r_{mt}, $\hat{u}_t$, and $\hat{\varepsilon}_t$ in table 9 may alleviate the statistical problems caused by non-trading of seats. Given the evidence of the Fisher effect in table 10, the sums of the coefficients on current and lagged r_{mt} or $\hat{u}_t$ in table 9 may be good estimates of the contemporaneous relationship

[18]Eugene Fama provided the data for the 1952–63 period and the Center for Research in Security Prices provided the data for the 1964–72 period.

Table 10

Tests of the 'Fisher effect' in NYSE market model regressions: $r_t = \hat{a} + [\beta_0 + \beta_1 L]r'_{mt} + [\vartheta_0 + \vartheta_1 L]r''_{mt} + \hat{\varepsilon}_t$.[a]

$\hat{a}$	β_0	β_1	ϑ_0	ϑ_1	$S(\hat{\varepsilon})$	R^2	$Q^*(12)$	S.R.($\hat{\varepsilon}$)
				August 1952 – *December* 1972				
−0.0024 (0.0062)	1.297 (0.1682)				0.095393	0.197	19.0	6.57
−0.0042 (0.0063)	1.280 (0.1678)	0.2830 (0.1676)			0.095031	0.206	19.2	6.44
−0.0009 (0.0064)			1.079 (0.1774)		0.099155	0.132	22.1	6.55
−0.0042 (0.0064)			1.037 (0.1745)	0.5660 (0.1745)	0.097267	0.168	22.8	6.45
				August 1952 – *December* 1962				
0.0047 (0.0075)	0.9525 (0.1919)				0.082666	0.167	11.0	5.67
0.0043 (0.0077)	0.9518 (0.1926)	0.0524 (0.1922)			0.082979	0.167	11.8	5.63
0.0053 (0.0077)			0.8766 (0.2048)		0.084493	0.130	13.1	5.56
0.0041 (0.0078)			0.8603 (0.2059)	0.1745 (0.2059)	0.084590	0.135	12.6	5.52
				January 1963 – *December* 1972				
−0.0097 (0.0098)	1.759 (0.2861)				0.105532	0.243	18.7	5.74
−0.0124 (0.0097)	1.691 (0.2852)	0.5460 (0.2854)			0.104362	0.266	17.5	5.84
−0.0071 (0.0104)			1.313 (0.2983)		0.112391	0.141	16.5	5.69
−0.0122 (0.0100)			1.261 (0.2842)	1.0365 (0.2842)	0.106953	0.229	16.0	5.93

[a] r'_{mt} = continuously compounded return on Standard and Poor's index matched to dates of last seat trade in each month.
r''_{mt} = continuously compounded return on Standard and Poor's index measured on the last day of each month.

between unexpected changes in seat prices and unexpected changes in stock prices or share trading volume. From this point of view, the elasticity of seat prices with respect to stock prices is much greater than the elasticity of seat prices with respect to unexpected share volume. This is a plausible result since the simple model in (5) has stock prices affecting revenue alone while share volume affects both revenue and costs.

This evidence based on the synchronous measurement of seat prices and stock prices suggests that the significance of the coefficients of lagged variables in section 4 can be explained by the fact that stock exchange seats are not traded on the last day of each month. The lagged relationships do not represent slow reaction of the market for seats to new information about the value of seats.

6. Relationship of seat price changes to returns on securities brokers' stocks

The prices of stock exchange seats should reflect the profitability of gaining access to the centralized secondary market on the floor of the exchange at a reduced cost. However, securities brokers may also earn profits from activities which are unrelated to the transactions which occur on the exchange floor. In order to get a measure of the extent to which seat prices reflect the overall profitability of exchange members we correlate the changes in NYSE seat prices with the returns on an equally-weighted portfolio of the stocks of NYSE members during the July 1971 – June 1975 period.

The New York Stock Exchange allowed its member firms to issue securities to the public for the first time in May 1970. Since early 1972 many of the largest brokerage firms have had their common stock traded on the NYSE or the ASE. For the purpose of this analysis we use the daily returns to the stocks of nine brokerage firms for the period from July 27, 1971 to June 30, 1975 from the Daily Master File of the Center for Research in Security Prices (CRSP) of the University of Chicago Graduate School of Business. The continuously compounded returns on an equally-weighted portfolio of brokers' stocks, r_{Bt}, are computed for the intervals between NYSE seat trades during this period.[19] The risk of this portfolio of brokers' stocks relative to the value-weighted portfolio of NYSE stocks from the CRSP Daily File is estimated using a market model regression for the 201 observations corresponding to seat trades,

$$r_{Bt} = \underset{(0.0030)}{-0.0019} + \underset{(0.1212)}{1.720} \cdot r_{mt} + \hat{e}_t, \tag{14}$$

$$R^2 = 0.503, \quad S(\hat{e}) = 0.04189, \quad Q^*(12) = 23.5,$$

[19]The equally-weighted portfolio included all of the stocks for which returns data are available on each day. After July 18, 1972, all nine stocks are always included in the portfolio. The companies whose securities are included in the portfolio are: Merrill, Lynch; Reynolds; Shearson, Hayden-Stone; Donaldson, Lufkin and Jenrette; A.G. Edwards; Dean Witter; Paine, Webber; Bache; and E.F. Hutton.

where the standard errors are in parentheses. The estimate of the risk of this portfolio, the coefficient of r_{mt}, is similar to the estimates of the risk of seats in table 10. The comparable regression of changes in the logs of seat prices on the value-weighted market portfolio over the July 1971 – June 1975 period is

$$\begin{aligned} r_t = \; & -0.0027 + 1.197 \; r_{mt} + \hat{e}_t, \\ & \;(0.0055) \quad (0.2267) \end{aligned} \tag{15}$$

$$R^2 = 0.123, \quad S(\hat{e}) = 0.07834, \quad Q^*(12) = 18.2,$$

which indicates that seats were somewhat less risky than brokers' stocks during this period. Thus, since brokers' stocks and stock exchange seats have similar levels of risk in a well-diversified portfolio they should have similar levels of expected return in the context of the capital asset pricing model.

The similarity of (14) and (15) in terms of the estimates of the slope and intercept is reflected in the correlation between r_t and r_{Bt} of 0.31 during the July 1971 – June 1975 period. The variance of r_{Bt} is lower than the variance of r_t because the former is the return on a portfolio of nine stocks while the latter is a single asset, so the proportion of variance explained, R^2, is higher and the standard error of the regression, $S(\hat{e})$, is lower in (14) than in (15).

Thus, it seems that NYSE seat prices may be a good proxy for aggregate long-run NYSE profitability, at least during the 1971–75 period for which data on the prices of brokers' stocks are available. This is reassuring since seat prices are the only data which are available for long periods of time which reflect the prosperity of securities brokers. There is also evidence in eqs. (14) and (15) that the activities of securities brokers are relatively risky in the framework of the capital asset pricing models of Sharpe, Lintner or Black.

7. Conclusions and suggestions for future research

This analysis of the time series behavior of the prices of stock exchange seats has shown that these prices behave much like the prices of individual NYSE-listed common stocks. Seat prices seem to follow a multiplicative random walk which is consistent with an efficient market in which expected returns are constant. The evidence that rates of change in seat prices may be normally distributed, at least over periods when the dispersion of the rates change is relatively constant, implies that the prices of stock exchange seats, conditional on last period's price, are distributed lognormally.

A simple model for the profitability of the securities brokerage industry is used to develop a time series regression model for the determination of changes in seat prices. Evidence from monthly data for the 1926–72 period suggests that unexpected percent changes in stock prices and share trading volume are important determinants of unexpected percent changes in seat prices. The absence of any significant effect of expected percent changes in share volume and the relatively small effects of lagged unexpected percent changes in stock prices and share volume indicate that the market for seats adjusts quickly to impound the effects of new information into seat prices.

The fact that seats are not sold on the last day of every month may cause spurious relationships between changes in seat prices and lagged changes in stock prices, share volume, or any other variable which affects the price of seats. In fact, there are some time periods when the regression models relating seat prices to stock prices and share volume had significant lagged relationships. In general, the American Stock Exchange data are more susceptible to this problem, especially in the 1966–72 period, but this would be predicted if the lagged relationships are caused by the 'Fisher effect' since ASE seats are traded less actively than NYSE seats. The evidence from the entire series of NYSE seat transactions over the 1952–72 period suggests that the significance of the lagged variables in the regression models of section 4 may be solely attributable to the nontrading phenomenon.

Thus, we have shown that the behavior of stock exchange seat prices is consistent with an efficient market for seats and constant expected returns to seatholders. We have also shown that brokers perceive *unexpected* changes in stock prices and share volume as important new information about the value of seats which is quickly assimilated by the market for seats. This relationship between seat prices and stock prices can be used to infer the riskiness of seats and, therefore, the expected return to a seat in the context of the capital asset pricing model of Sharpe–Lintner–Black and others. In fact, the estimated relative risk of NYSE seats is quite similar to the estimated relative risk of the common stocks of some large NYSE brokerage firms over the 1971–75 period.

One potential use of the models developed in this paper would be to analyze the effects of public regulation on the profitability of securities brokers. As Doede (1967) notes, if the expected net value of NYSE assets conditional on dissolution is small the value of seats will reflect the discounted flow of rents expected by member brokers as a result of the fact that it is relatively inexpensive to trade securities in a centralized market and stock exchange members have cheap access to that market. Thus, changes in regulations which affect brokers' profitability would be directly reflected in seat prices. Schwert (1975) uses the models developed in section 4 to analyse the effects of many important changes in regulation by the Securities and Exchange Commission. One of the major findings of that study is that seat prices fell unexpectedly by about fifty percent in the month when the Securities and Exchange Act of 1934 was introduced into Congress and there is no evidence that this capital loss was ever recovered (see fig. 1 for visual evidence of this event in March 1934).

References

Bank and Quotation Record (William B. Dana, New York).

Black, F., 1972, Capital market equilibrium with restricted borrowing, Journal of Business 45, 444–454.

Blattberg, R.C. and N.J. Gonedes, 1974, A comparison of the stable and student distributions as statistical models for stock prices, Journal of Business 47, 244–280.

Box, G.E.P. and G.M. Jenkins, 1970, Time series analysis (Holden-Day, San Francisco, CA).

Box, G.E.P. and D. Pierce, 1970, Distribution of residual autocorrelations in autoregressive-integrated-moving average time series models, Journal of the American Statistical Association 65, 1509–1526.

Clark, P.K., 1973, A subordinated stochastic process model with finite variance for speculative prices, Econometrica 41, 135–156.

Cootner, P., ed., 1964, The random character of stock market prices (MIT Press, Cambridge, MA).

David, H.A., H.O. Hartley and E.S. Pearson, 1954, The distribution of the ratio, in a single normal sample, of range to standard deviation, Biometrika 41, 482–493.

Doede, R.W., 1967, The monopoly power of the New York Stock Exchange, unpublished doctoral dissertation (University of Chicago, Chicago, IL).

Fama, E.F., 1965a, The behavior of stock prices, Journal of Business 38, 34–105.

Fama, E.F., 1965b, Tomorrow on the New York Stock Exchange, Journal of Business 38, 285–299.

Fama, E.F., 1970, Efficient capital markets: A review of theory and empirical work, Journal of Finance 25, 383–417.

Fama, E.F., 1976, Foundations of finance (Basic Books, New York).

Granger, C.W.J., 1975, Some consequences of the valuation model when expectations are taken to be optimum forecasts, Journal of Finance 30, 135–145.

Ibbotson, R., 1975, Price performance of common stock new issues, Journal of Financial Economics 2, 235–272.

Jensen, M.C., 1972, Capital markets: Theory and evidence, Bell Journal of Economics and Management Science 3, 357–398.

Lintner, J., 1965, Security prices, risk, and maximal gains from diversification, Journal of Finance 20, 587–616.

Officer, R.R., 1973, The variability of the market factor of the New York Stock Exchange, Journal of Business 46, 434–453.

Pierce, D., 1972, Least squares estimation in dynamic-disturbance time series models, Biometrika 59, 73–78.

Schwert, G.W., 1975, Public regulation of national securities exchanges: A test of the capture hypothesis, Center for Mathematical Studies in Business and Economics Report no. 7546 (University of Chicago, Chicago, IL).

Sharpe, W.F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance 19, 425–442.