Short-Term Interest Rates as Predictors of Inflation: On Testing the Hypothesis that the Real Rate of Interest is Constant

By CHARLES R. NELSON AND G. WILLIAM SCHWERT*

In an innovative and provocative paper in this *Review*, Eugene Fama presents evidence which appears to be consistent with the joint hypothesis that the real rate of interest, ignoring taxes, is a constant and that the market for U.S. Treasury Bills is efficient in the sense of embodying rational expectations.[1] Those findings are strikingly at variance with a long list of previous studies which seemed to support the view that the real rate of interest varies over time and can be related to economic variables such as real output, monetary policy, and so forth.[2] The methodologies which lead to these contradictory conclusions are quite different. Previous studies had followed the lead of Irving Fisher by relating nominal interest rates to distributed lags on past rates of inflation.[3] These distributed lags are generally interpreted as approximations to the mar-

ket's expected rate of inflation and thus any remaining variation is attributed to variation in the real rate.[4] Fama's methodology, on the other hand, draws on the fact that the difference between the market interest rate and the *subsequently observed* rate of inflation, the *ex post* real interest rate, consists by definition of the *ex ante* real interest rate plus a pure forecasting error.[5] The hypothesis of market efficiency implies that these forecasting errors must be serially random. Thus, observing *ex post* real rates is equivalent to observing *ex ante* real rates with random measurement error. These errors of course confound the problem of identifying variation in the *ex ante* real rate.

In this paper, we will argue that the relative magnitude of these measurement errors is such that the tests carried out by Fama were not powerful enough to reject the joint hypothesis that the *ex ante* real rate is a constant and expectations are rational. More powerful tests which are presented in this paper using the same data do lead to rejection of that hypothesis. Of course, rejection of the joint hypothesis could be interpreted

[1] See John Muth and Fama (1970) for definitions of "rationality" and "efficiency."

[2] The taxation of nominal income implies that if the before-tax real rate is constant, the after-tax real rate will vary inversely with the rate of inflation. The behavior of the after-tax rate is presumably more relevant from the viewpoint of the individual investor making consumption savings decisions.

[3] Richard Roll presents a bibliography of many of these studies. Roll also argues that efficiency in markets for goods (with no costs of storing goods or shifting production through time) would imply that inflation rates be randomly distributed with mean zero, but shows that to the contrary actual inflation rates exhibit substantial autocorrelation.

[4] As John Rutledge (pp. 17–21) has pointed out, Fisher's interpretation of his distributed lags placed considerable emphasis on the indirect effects of inflation on interest rates rather than on the formation of expectations by extrapolation.

[5] Fama's methodology corresponds to Reuben Kessel's methodology for studying the behavior of liquidity premiums in the term structure of interest rates. In the present context, subtracting the observed inflation rate from the market interest rate removes the expected inflation rate, leaving the *ex ante* real rate and a random forecasting error. Kessel subtracted observed spot interest rates from corresponding past forward rates thereby removing the expected spot rate and leaving the liquidity premium and a random forecasting error. Under the implicit assumption of rational expectations, Kessel studied the relationship of liquidity premiums to the level of interest rates, but he did not claim to have tested that assumption.

as an indication that the *ex ante* real rate is variable or that the market is inefficient, or both. In fact, neither market efficiency nor the constancy of the real rate constitutes in itself a testable hypothesis. If we are willing to *assume* that the market is efficient, then our results suggest that variation in the real rate is of the magnitude suggested by previous studies in the Fisher tradition.[6]

I. Tests Based on Autocorrelation in the Ex Post Real Rate

Fama examines the sample autocorrelations of the *ex post* real rate as one test of his joint hypothesis. As mentioned above, the *ex post* real rate (*EPRR*) consists by definition of the *ex ante* real rate (*EARR*) plus a pure forecasting error which will be serially uncorrelated if the market is efficient. In such a market, any autocorrelation in the *EPRR* can be attributed to autocorrelation in the *EARR*. For the monthly data consisting of 1-month Treasury Bill yields used by Fama and 1-month rates of change in the Consumer Price Index (*CPI*), the sample autocorrelations for lags one through twelve computed over the period January 1953 through July 1971, are given in Table 1.[7] The autocorrelations are generally small relative to their asymptotic standard error (equal to $1/\sqrt{n}$) with the exception of the seasonal autocorrelation at lag 12 months. The Box-Pierce Q-statistic, a measure of overall autocorrelation, is fairly large and significant at the .10 level, but this is due primarily to the seasonal coefficient. Autocorrelation in the *EPRR* need not be due to market inefficiency; rather, as Fama points out, it could be due to systematic measurement errors in the Consumer Price Index or autocorrelation in the *EARR*. The fact that the sample autocorrelations of the *EPRR* are small was interpreted by Fama as an indication of market efficiency

[6] For example, see the calculated real rate series by William Yohe and Denis Karnosky.

[7] We use the more familar rate of inflation, the natural logarithm of the price relative, instead of the discrete percent change in purchasing power used by Fama. Also, our estimated autocorrelations are computed as correlation coefficients whereas Fama computed regression coefficients.

TABLE 1—SAMPLE AUTOCORRELATIONS OF ONE-MONTH *Ex Post* REAL RATE, JANUARY 1953–JUNE 1971

Lag	Autocorrelation	Lag	Autocorrelation
1	.10	7	−.08
2	.12	8	.04
3	−.02	9	.10
4	−.01	10	.09
5	−.02	11	.03
6	−.01	12	.18

Note: Standard Error = .07; Box-Pierce $Q = 19.4$ (x^2 with 12 degrees of freedom).

and constancy of the *EARR*. However, lack of serial correlation in the *EPRR* is also consistent with variation in the *EARR* which is purely random in nature and also with market inefficiency in the form of forecast errors which are larger than necessary given available information. What we wish to demonstrate here is that even the rather low autocorrelation observed in the *EPRR* could in fact be indicative of rather strong autocorrelation and sizable variation in the *EARR* since the latter is being overlaid with forecast errors when we observe the former.

Suppose that the *EARR* rather than being constant is a stochastic process with first-order serial correlation coefficient ϕ. Denoting the market's forecasting error for the rate of inflation by ϵ_t, the *EARR* by i_t, and the EPRR by r_t, we have

$$(1) \qquad r_t \equiv i_t - \epsilon_t$$

It is easy to show that in an efficient market the first-order serial correlation coefficient for the *EPRR* will be related to ϕ by

$$(2) \qquad \text{Corr}\,(r_t, r_{t\pm1}) = \frac{\phi\sigma_i^2}{\sigma_i^2 + \sigma_\epsilon^2}$$

where σ_i^2 and σ_ϵ^2 denote the variances of the *EARR* and the forecast error, respectively. It should be clear that first-order autocorrelation in the *EPRR* may be considerably less than ϕ if the variance of forecast errors is large relative to the variance of the *EARR*.

It is interesting to consider what combinations of autocorrelation and variance for the *EARR* are consistent with low auto-

TABLE 2—VALUES OF ϕ AND σ_w^2 AND CORRESPONDING VARIANCES AND STANDARD DEVIATIONS FOR THE *EARR* AS AN ANNUAL PERCENTAGE RATE CONSISTENT WITH $Corr(r_t, r_{t+1}) = .10$ and $\sigma_\epsilon^2 = 4.32$

ϕ	Variance of EARR, σ_i^2	Standard Deviation of EARR, σ_i	σ_w^2
.4	1.44	1.20	1.21
.5	1.08	1.04	.806
.6	.864	.924	.547
.7	.720	.852	.374
.8	.619	.780	.230
.9	.547	.732	.102
.95	.508	.708	.049
.99	.485	.696	.010
.999	.481	.696	.001

Note: The statistic σ_w^2 is computed under the assumption: $i_t = \phi i_{t-1} + w_t$.

correlation in the *EPRR*, say the .10 observed during the sample period. To do this we require an estimate of σ_ϵ^2. An upper bound estimate of σ_ϵ^2 would be the variance of the *EPRR*, 5.18 on an annualized percentage basis, since that estimate would attribute all variation in the *EPRR* to the forecast error. Estimates derived in Section III under less restrictive assumptions suggest that a conservative estimate (in the sense of yielding smaller values of ϕ and σ_i^2) for purposes of illustration would be 4.32 percent on an annual basis. Corresponding values of ϕ and of the variance and standard deviation of the *EARR* in annualized percentage terms are given in Table 2. It is apparent that low autocorrelation in the *EPRR* may be compatible with substantial variation and autocorrelation in the *EARR*. For example, values of ϕ of .99 or .999 which would imply behavior approaching nonstationarity are also consistent with a standard deviation in the *EARR* of about .7 percent or a .95 probability interval of nearly 3 percent, assuming normality.

Even if the *EARR* were a random walk having no long-run mean and unbounded variance, we might very well observe sample autocorrelations as low as those in Table 1. Denoting the hypothetical random steps in the *EARR* by w_t, so that $i_t = i_{t-1} + w_t$, it is easy to show that a random walk in the *EARR* would imply that the *EPRR* is

generated by the process

$$(3) \qquad r_t = r_{t-1} + w_t - \epsilon_t + \epsilon_{t-1}$$

which could be thought of as a random walk with an autocorrelated disturbance. The *theoretical* autocorrelations for r are undefined (as they are for i) but *sample* autocorrelations are of course readily computed for any finite data series and their magnitude will depend on the autocorrelation properties of the moving average ($w_t - \epsilon_t + \epsilon_{t-1}$). The analysis given by George Box and Gwilym Jenkins implies that the sample autocorrelations of r_t will tend to be quite small if the variance of w_t is small relative to the variance of the forecasting errors ϵ_t.[8] In particular, a value of .011 for the variance of w_t would be implied by our estimate of σ_ϵ^2 and the observed sample first-order autocorrelation in r. To interpret this, note the last column of Table 2 which gives the variance of w_t, denoted σ_w^2, in the stationary process $i_t = \phi i_{t-1} + w_t$, $|\phi| < 1$, for the accompanying values of ϕ and σ_i^2. The value of σ_w^2 computed from the formula given by Box and Jenkins under the random walk assumption is in fact larger than the values in Table 2 for the cases $\phi = .99$ and $.999$.

We conclude in this section that tests of Fama's hypothesis based on sample autocorrelations of the *EPRR* will have little power against alternatives which specify economically plausible variation and autocorrelation in the *EARR* or even the alternative hypothesis of a random walk in the *EARR*.

II. Regression Tests

Fama also presented tests of the joint hypothesis of a constant real rate and an efficient market based on regressions of realized inflation rates ρ_t, on the market interest rate R_t, and past rates of inflation. If $\hat{\rho}_t$ represents a piece of information about ρ_t which is available to the market at the beginning of period t, for example, a past

[8] Box and Jenkins (pp. 200–01) evaluate the ratio of the expectation of the sample autocovariance at lag one to the expectation of the sample variance, both conditional on the initial observation in the sample, for a process similar to (3).

rate of inflation, then the regression of ρ_t on R_t and $\hat{\rho}_t$ would yield a nonzero coefficient for $\hat{\rho}_t$ only if the market were either inefficient in its use of available information or if the predictive ability of R_t were distorted by underlying variation in the *EARR*. Fama chose ρ_{t-1} as a particular $\hat{\rho}_t$ and found that the coefficient of ρ_{t-1} was small and not significant. The power of such a test will be low, however, if the $\hat{\rho}_t$ chosen contains little information about ρ_t.

In order to provide a more powerful test of the joint hypothesis of market efficiency and the constancy of the *EARR*, we use the past rates of inflation to construct a $\hat{\rho}_t$ which is an optimal extrapolative predictor using the methodology of Box and Jenkins. While this approach will not generally yield a fully rational forecast of ρ_t, it will generally be a more efficient predictor than ρ_{t-1}.[9]

The weights assigned to R_t and $\hat{\rho}_t$ in a composite prediction of ρ_t can be related to measures of market efficiency and variation in the *EARR*. Define u_t as the component of information embodied in the market forecast but not in the extrapolative predictor, and v_t as the component of information embodied in the extrapolative predictor but ignored by the market. These random variables have variances σ_u^2 and σ_v^2, respectively, and are uncorrelated with each other. The probability limits of the weights assigned to R_t and $\hat{\rho}_t$ in the composite predictor of ρ_t are approximately $\sigma_u^2/(\sigma_u^2 + \sigma_v^2 + \sigma_i^2)$ and $(\sigma_v^2 + \sigma_i^2)/(\sigma_u^2 + \sigma_v^2 + \sigma_i^2)$, respectively, if the *EARR* is uncorrelated with the anticipated rate of inflation. Thus, the relative weight assigned to $\hat{\rho}_t$ will be greater than zero if variation in the *EARR* (σ_i^2) is nonzero or if the market ignores any information available from past inflation rates (measured by σ_v^2).[10]

The autocorrelation structure of the *CPI* monthly inflation series for January 1953 through July 1971 suggests that the series may reasonably be represented as a first-order moving average process in its first differences, implying nonstationary behavior in the rate of inflation. The estimated model for the 2/53–7/71 period is

$$(4) \qquad (1 - B)\rho_t = .0222 + (1 - .894B)e_t$$
$$\qquad\qquad (.0179) \qquad\quad (.029)$$
$$\hat{\sigma}_e = 2.408$$

where e_t is a sequence of residuals which are the one-step-ahead forecast errors for this model and B is the lag operator. The forecast $\hat{\rho}_t$ of ρ_t implied by this model may be written apart from a constant as

$$(5) \qquad \hat{\rho}_t = .11\rho_{t-1} + (.89)(.11)\rho_{t-2} + \dots$$
$$\qquad + (.89)^j(.11)\rho_{t-j-1} + \dots$$
$$= \sum_{j=0}^{\infty} \theta^j(1 - \theta)\rho_{t-j-1}$$

where $\theta = .89$. Note that the weight given to ρ_{t-1} in this forecast is very small; interestingly, it is the same as the regression coefficient of ρ_{t-1} in Fama's regressions of ρ_t on R_t and ρ_{t-1} for monthly data.[11] Also note that the sample standard deviation of the forecast errors e_t is larger than our estimate of the standard deviation of market forecast errors, the residuals from the regression of ρ_t on R_t in Table 3, which is consistent with the hypothesis that the market utilizes information beyond that available from past inflation rates alone. Reestimation of (4) over the subperiods 1/53–2/59, 3/59–7/64, and 8/64–7/71 revealed little variation in the estimates of the moving average parameter θ. This model can be used to predict either the level ρ_t, or the change $\rho_t - \rho_{t-1}$ in the rate of inflation; in either case, the one-step-ahead prediction errors are the disturbances e_t.

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[9] Rutledge (pp. 57–61) has carried this reasoning one step further in the context of a model in which expectations of inflation depend on past money growth rates as well as past inflation rates. Regressions of the *EPRR* associated with forward Treasury Bill yields on these variables led Rutledge to conclude that the joint hypothesis of rational expectations and constancy of the (forward) real rate could not be rejected.

[10] It should be noted that serially correlated measurement errors in the *CPI* might be predicted by $\hat{\rho}_t$

but ignored by R_t, since an efficient market should ignore these measurement errors in setting interest rates. This possibility is observationally equivalent to the possibility that the market ignores information contained in past inflation rates.

[11] See Fama (1975), Table 4, p. 276, where the regression coefficient for ρ_{t-1} is given as .11 for the 1/53–7/71 sample period.

TABLE 3

α	β	γ	$\hat{\sigma}_e$	R^2	D.W.
A. Composite Predictors of the Rate of Inflation: 2/53–7/71					
−.775	.969		2.347	.292	1.81
(.358)	(.102)				
−.641	.651	.383	2.322	.310	1.93
(.359)	(.165)	(.158)			
B. Composite Predictors of the Change in the Rate of Inflation: 2/53–7/71					
−.774	.889		2.333	.458	2.07
(.167)	(.065)				
−.546	.633	.317	2.320	.466	2.03
(.206)	(.152)	(.170)			

Note: The prediction equations are:
A. $\rho_t = \alpha + \beta R_t + \gamma \hat{\rho}_t + e_t$
B. $(\rho_t - \rho_{t-1}) = \alpha + \beta(R_t - \rho_{t-1}) + \gamma(\hat{\rho}_t - \rho_{t-1}) + e_t$

(Standard errors in parentheses.)

Regressions of ρ_t on R_t and $\hat{\rho}_t$ for the 2/53 to 7/71 period in Table 3 indicate that the extrapolative predictor has a large and significant weight in the composite predictor for ρ_t. Since there is evidence that ρ_t is not stationary, we also estimate regressions which are designed to predict the change in the rate of inflation $\rho_t - \rho_{t-1}$, using predictors of the change: $R_t - \rho_{t-1}$ and $\hat{\rho}_t - \rho_{t-1}$. Again, the coefficient of the extrapolative predictor is large though not as strongly significant as for the level of the rate of inflation.

These composite prediction tests are subject to criticism on the grounds that the extrapolative predictors are computed using time-series models estimated over the entire sample period and thus possibly use information not fully available to the market. This is not likely to be important since estimates of the parameters of the time-series models are quite stable when estimated over subperiods. As a check on the sensitivity of the results we computed $\hat{\rho}$ sequentially, using data for 1/43–12/52 to calculate forecasts for 1953, then data for 1/44–12/53 to calculate forecasts for 1954, and so forth.[12] We found that the differences in the predic-

[12]This is essentially the strategy followed by Patrick Hess and James Bicksler in their analysis of Fama's regression tests.

tive regressions were too small to alter our basic conclusions. Ideally, the composite weights should be estimated simultaneously with the parameters of the time-series model by specifying a dynamic regression (transfer function) model which includes pure interest rate and pure time-series prediction as special cases. The resulting point estimates and their standard errors would reflect appropriately the information contained in the data. Taking this approach we embedded the interest rate in the time-series model for inflation and obtained the following results for the 2/53–7/71 period:

$$(6) \quad (1 - B)\rho_t = .0199 + .577(1 - B)R_t$$
$$(.0211) \quad (.262)$$
$$+ (1 - .876B)e_t$$
$$(.032)$$
$$\hat{\sigma}_e = 2.430$$

If the coefficient of $(1 - B)R_t$ is zero, the model reduces to the pure extrapolative model; on the other hand, if the coefficient of e_{t-1} is unity, the model reduces the regression of ρ_t on R_t. Both of these polar cases are rejected by the data.[13]

These composite prediction regressions strongly suggest that the EARR is not constant since, as we have seen, in an efficient market "naive" predictors such as $\hat{\rho}_t$ should add nothing to the predictive power of the market interest rate. These results are consistent, however, with the hypothesis that part of the variation in R is due to the non-predictive variation of the EARR. Our results also suggest that the market draws on information beyond that available from past inflation rates ($\sigma_u^2 > 0$) since the weight given to the interest rate is large and significant.

Similar results are obtained when the time-series model is expanded to account for seasonality which is present in the CPI.[14] Since this seasonal autocorrelation is

[13]The observant reader may have noted that $\hat{\sigma}_e$ is larger in (6) where R_t is included than in the pure time-series model (4) where R_t does not appear. This is due to a difference in procedures for computing residuals in non-linear least squares which involved "backforecasting" presample data in the case of (4) but not in (6).
[14]The seasonal model is a multiplicative seasonal moving average model with seasonal and ordinary dif-

exploited by the model to increase the predictive power of $\hat{\rho}$, it is not surprising that the weight given $\hat{\rho}$ in composite predictions increases from the values of .383 and .317 in Table 3 to .448 and .627, respectively, with roughly corresponding reductions in the weights given to R. When R is embedded in the seasonal time-series model, it continues to exert a significant influence. Some of the seasonality in the CPI may be due to the fact that prices of certain items are not sampled every month. For this reason we feel it is more conservative to emphasize the results which do not depend on predicting seasonal variation.

III. Identification of Estimates of the Variance of the Ex Ante Real Rate of Interest

Since the joint hypothesis of market efficiency and constancy of the $EARR$ is rejected by the composite prediction tests, it is a matter of some interest to identify an estimate of the variance of the $EARR$ in terms of observable variables. We approach this problem by examining the relationships linking the variances and covariances of observed variables to those of the unobserved components of those variables including the $EARR$. We begin with the definitions

$$(7) \qquad R_t = \rho_t^* + i_t$$
$$\rho_t = \rho_t^* + \epsilon_t$$

where R_t is the observed nominal interest rate, ρ_t^* is the unobserved market forecast of the inflation rate ρ_t, i_t is the unobserved $EARR$ as before, and ϵ_t is the market forecast error as before. The variances of R and ρ and their covariances can be estimated from data. From these we would like to solve for the variances of i, ϵ, ρ^*, and their covariances. For a solution to be possible, we need to impose enough restrictions to reduce the number of unknowns to three. Under the *assumption* that the market is efficient, we can eliminate the covariances be-

tween i and ϵ and between ρ^* and ϵ, since both i and ρ^* represent *ex ante* information. If we are willing to assume further that the covariance between ρ^* and i is zero (no "Mundell Effect"), then the covariance between R and ρ is simply the variance of ρ^* and thus

$$(8) \qquad \hat{\sigma}_i^2 = \hat{\sigma}_R^2 - \hat{\sigma}_{R\rho}$$
$$\hat{\sigma}_\epsilon^2 = \hat{\sigma}_\rho^2 - \hat{\sigma}_{R\rho}$$

The same relations hold with regard to variances and covariances of changes in the rate of inflation $(\rho_t - \rho_{t-1})$ and predicted changes in the rate of inflation $(R_t - \rho_{t-1})$ and $(\rho_t^* - \rho_{t-1})$. Since there is some doubt about the stationarity of ρ_t and R_t and therefore about the existence of variances and covariances, computations in terms of changes in the rate of inflation and $(R_t - \rho_{t-1})$ may be preferable. Sample moments for these variables from the data used by Fama imply

$$(9) \qquad \hat{\sigma}_i^2 = .642$$
$$\hat{\sigma}_\epsilon^2 = 4.85$$

It is interesting to note that this estimate of σ_i^2, the variance of the $EARR$, is quite close to those presented in Table 2 which are associated with a fairly strongly autocorrelated $EARR$.

In a well-known paper, Robert Mundell has argued that an increase in the anticipated rate of inflation will be accompanied by a fall in the *ex ante* real rate. A full interpretation of Mundell's result in a dynamic context goes beyond the scope of this paper, but it does suggest that the covariance over time between the expected rate of inflation or expected changes in the rate of inflation and the *ex ante* real rate may be nonzero. It is clear from the above analysis that additional information must be added if an estimate of this covariance is to be identified. Additional information is available from the variance of $(\hat{\rho}_t - \rho_{t-1})$ and its covariance with $(R_t - \rho_{t-1})$ and $(\rho_t - \rho_{t-1})$, although the covariance of $(\hat{\rho}_t - \rho_{t-1})$ with i, $(\rho_t^* - \rho_{t-1})$ and ϵ will be additional unknowns. Market efficiency implies that the third of these covariances is zero, and if we are willing to assume that

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ferencing. The moving average parameters are both about .9, so that the seasonal and nonseasonal parts of the model are very similar. The parameter estimates were again very stable over subperiods. We benefited from discussions with Charles Plosser on seasonality in the CPI.

the covariance between $(\hat{\rho}_t - \rho_{t-1})$ and i is the same as that between $(\rho_t^* - \rho_{t-1})$ and i, then it is easy to show that solution is feasible. The resulting estimates are

$$(10) \quad \hat{\sigma}_{(\rho_t^* - \rho_{t-1}),i} = \hat{\sigma}_{(\hat{\rho}_t - \rho_{t-1}),i} = .058$$
$$\hat{\sigma}_i^2 = .585$$
$$\hat{\sigma}_\epsilon^2 = 4.91$$

The estimated covariance between expected inflation and the *EARR* is positive and so small that it causes only a slight change in $\hat{\sigma}_i^2$ and $\hat{\sigma}_\epsilon^2$ relative to equations (9).

This analysis of the variances and covariances of the unobservable components of the interest rate and inflation can be extended to the interpretation of regressions of ρ_t on R_t as reported by Fama. As Fama noted, a slope coefficient of unity in these regressions would be consistent with the hypothesis that the *EARR* is constant. Since the estimated coefficient in this regression (Table 3, Panel A) is .969 with a standard error of .102, the hypothesis seems to be supported by the monthly data. We have argued previously in this paper that the rate of inflation is nonstationary; thus for purposes of discussing probability limits it is more appropriate to work with changes in the rate of inflation and predicted changes. The slope in the change regression (Table 3, Panel B) is .889 with a standard error of .065, so this alternative regression would seem to cast doubt on the constant *EARR* hypothesis. The probability limit of this slope is the ratio of the covariance between $(\rho_t - \rho_{t-1})$ and $(R_t - \rho_{t-1})$ over the variance of $(R_t - \rho_{t-1})$ which is equivalent to

$$(11) \quad plim \, \hat{\beta}_{(\rho_t - \rho_{t-1}),(R_t - \rho_{t-1})}$$
$$= \frac{\sigma_{(\rho_t^* - \rho_{t-1})}^2 + \sigma_{(\rho_t^* - \rho_{t-1}),i}}{\sigma_{(\rho_t^* - \rho_{t-1})}^2 + \sigma_i^2 + 2\sigma_{(\rho_t^* - \rho_{t-1}),i}}$$

This expression will deviate from unity if variation in the *EARR* is nonzero. The estimated slope of .889 can be interpreted, then, as being consistent with the small positive estimate of $\sigma_{(\rho_t^* - \rho_{t-1}),i}$ given in (10), and variation in the *EARR* which is small *relative* to the variation in the predicted change in the rate of inflation.

These results change in an interesting way if the $\hat{\rho}$ used in the calculations is generated by the seasonal time-series model. In this case we find

$$(12) \quad \hat{\sigma}_{(\rho_t^* - \rho_{t-1}),i} = \hat{\sigma}_{(\hat{\rho}_t - \rho_{t-1}),i} = -1.03$$
$$\hat{\sigma}_i^2 = 1.67$$
$$\hat{\sigma}_\epsilon^2 = 3.82$$

Thus there does seem to be a Mundell Effect in seasonal variation.[15] This negative covariance between the *EARR* and the expected change in the rate of inflation is sufficient to increase substantially the estimate of the variance of the *EARR* implying a .95 probability range of about 5 percent on an annual basis.

All of these estimates of the variance of the *ex ante* real rate of interest are consistent with the results of the previous sections of the paper. Under a variety of assumptions about the relationships among the unobservable components of the market interest rate and the rate of inflation, we estimate the standard deviation of the *ex ante* monthly real rate to be between .7 and 1.3 expressed as an annualized percentage rate.

IV. Conclusion

As Fama noted, it is not possible to test the efficiency of the market for U.S. Treasury Bills without some model for the behavior of the *ex ante* real rate of interest. Conditional on the assumption that the *EARR* is constant, Fama used the sample autocorrelation function of the *ex post* real rate and the regression relationship of rates of inflation on market interest rates and the prior rate of inflation to test market efficiency. However, those tests may not have been powerful enough to lead to a rejection of the joint hypothesis of market efficiency and constancy of the *EARR* even if the *EARR* varies as much as has been indicated by previous studies.

We have shown that the autocorrelation

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[15] Hess and Bicksler test for and find a negative relationship between the level of inflation and the *EARR*. We interpret Mundell's analysis as one which applies to short-run shifts in expectations rather than long-run movements in the level of expectations such as have occurred over the postwar period.

function of the *ex post* real rate of interest may be quite close to zero at all lags, even if the *ex ante* real rate varies substantially and is highly autocorrelated, because of the relatively large variance of errors in expectations of inflation. In fact, the autocorrelations of the *EPRR* may be small even if the *EARR* is a nonstationary stochastic process.

By examining the time-series properties of rates of inflation computed from the *CPI*, we have shown that *individual* past inflation rates contain very little information about future rates of inflation, suggesting that Fama's test based on the regression of the inflation rate on the market interest rate and the prior inflation rate is not a powerful test. To increase the power of the regression test we use the time-series properties of the rate of inflation to construct an optimal predictor of inflation based on the past history of inflation rates. The coefficient of the time-series predictor is large and significant in a composite prediction regression equation which also includes the market interest rate. This result would only occur if the market were inefficient in assimilating information contained in past inflation rates, or if variation in the *EARR* distorts the predictive variation in the market interest rate, or both. Thus, by making more efficient use of the information about future inflation which is contained in past inflation rates, we are able to reject Fama's joint hypothesis.

In order to get estimates of the magnitude of the variability of the *ex ante* real rate, we *assume* market efficiency and use the relationships among the unobservable components of the market interest rate and the observed inflation rate to identify estimates of the variance of the *EARR*. These estimates are comparable to those which have been derived by others working in the Fisher tradition and they indicate substantial variability in the monthly *EARR*.

There are many important issues related to measurement errors in the *CPI*. Certain kinds of errors can yield implications which are observationally equivalent to market inefficiency or variation in the *EARR*. Other types of measurement errors could have opposite effects on tests of Fama's hypothe-

sis; for example, since the *CPI* is collected in mid-month the beginning-of-month interest rate has a two week lead on the inflation rate computed from the *CPI*. However, a full treatment of the effects of measurement error in the *CPI* is beyond the scope of this paper. In all of our analysis we have taken the data at face value and focused on the statistical issues raised by Fama's tests.[16]

Finally, while we are obliged to reject Fama's joint hypothesis, we do feel that Fama made an important methodological contribution to the literature on interest rates and inflation by shifting attention from regressions of interest rates on past inflation rates to relationships between interest rates and subsequently observed inflation rates. Our analysis does suggest that expectations of inflation have accounted for most of the variation in short-term interest rates during the postwar period, and that those expectations embody significant information beyond that contained in past inflation rates alone.

[16]However, we do not agree with Fama (1975, p. 274) that the adequacy of the data should be judged, *ex post*, by the outcome of tests of his hypothesis.

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